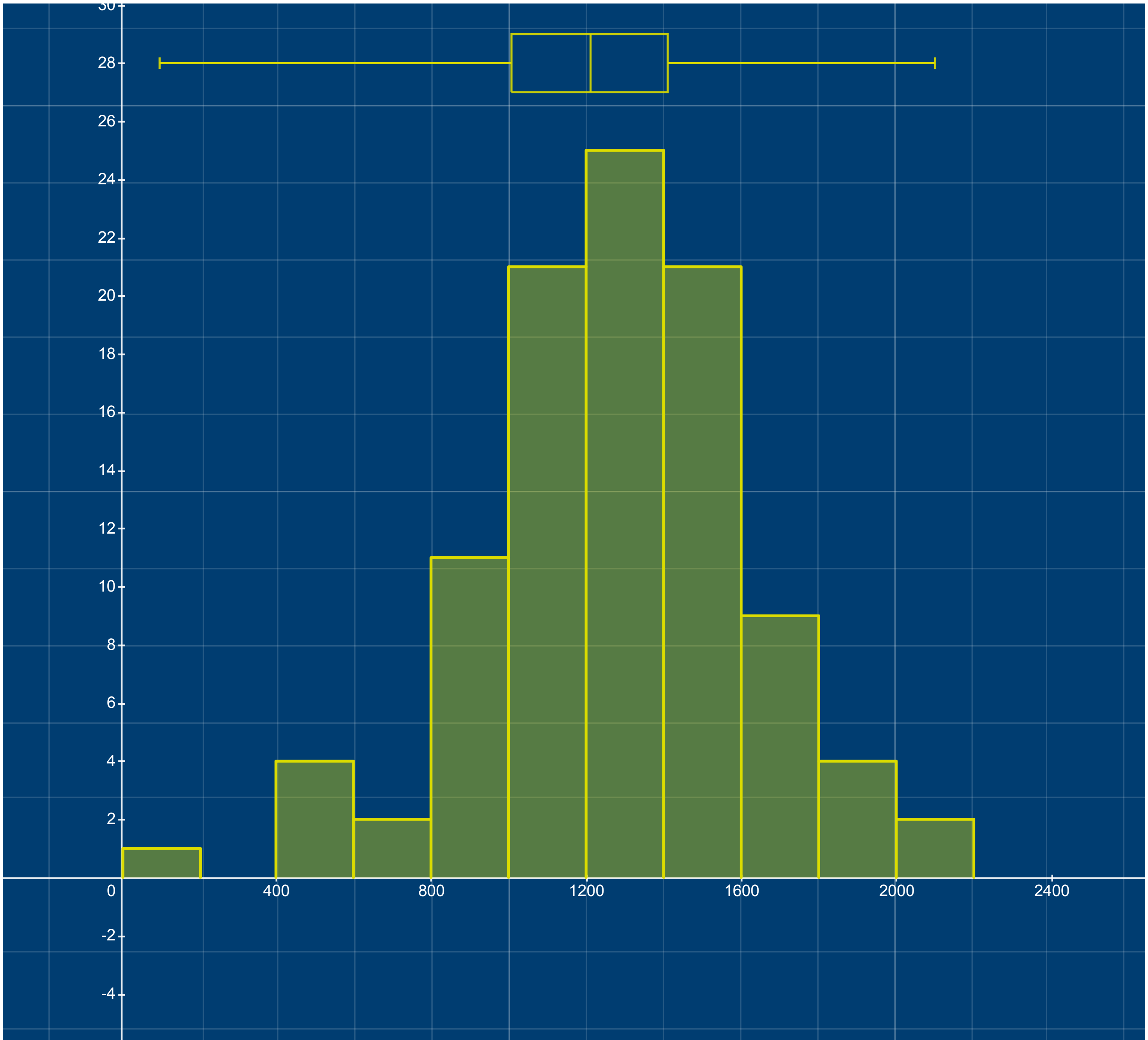


Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

TEACHER WORKBOOK 1

Unit 2

Table of Contents

Unit 2: Linear Equations and Inequalities

In middle school, students began building an understanding of how variables, expressions, equations, and inequalities could be used to represent quantities and relationships. Students also made connections among different kinds of representations—algebraic, verbal, tabular, and graphical. In this unit, students further develop their capacity to create, manipulate, interpret, and connect these representations and to use them for modeling.

In Lessons 1–4, students learn to think of equations as a way to represent constraints or limitations on quantities. (For instance, if the cost of food, f, and the cost of drinks, d, for a party add up to $80, the equation $f+d=80$ can be written to represent this constraint.) Students understand that solving equations means looking for values that satisfy the constraints and make the equations true. (For example, $f=53$ and $j=27$ could be a pair of solutions to $f+d=80$, but $f=50$ and $d=35$ could not be.)

Students then investigate, in Lessons 5 and 6, different ways to express the same relationship or constraint by analyzing and writing equivalent equations. They look at moves that can transform one equation to an equivalent equation, recognizing that these are the moves necessary to solve equations. The focus here is not only on identifying acceptable moves for solving, but also on explaining why these moves keep each subsequent equation true and maintain the solutions of the original equation.

Lessons 7 and 8 provide opportunities for students to expand on the work of writing and solving equations from middle school. Lessons 9 and 10 are Checkpoint Lessons. These lessons have three main purposes: 1. differentiated and small-group instruction; 2. opportunities for students to participate in various learning stations to refine and extend previous learning; and 3. the opportunity for students to complete the next unit's Check Your Readiness (CYR). Administering the CYR at this point in the unit allows plenty of time for the data to inform the next unit's instruction. Additionally, one of the learning stations explores the differences in wage earnings in the United States based on gender, race, and age.

In Lessons 11 and 12, students realize that some equations are more helpful than others, depending on the desired solution. In some equations, the quantity of interest is easy to pin down. In others, students may need to manipulate the equation and solve for a particular variable.

Lessons 13 and 14 are the first Mathematical Modeling lessons in Math 1. These lessons provide an opportunity for students to gain understanding of the modeling cycle and its use in making the link between the mathematics they learn in school and what they will encounter in college, career, and life. Students exploring Modeling Prompt #2 will have the opportunity to gather or research data to help answer a question that is important or interesting to them.

In the final part of the unit, Lessons 15–17, students rely on their understanding of equations to explore inequalities in one variable. They see that inequalities are a handy way to express constraints that involve an upper or lower limit, and can be satisfied by a range of values rather than a single value. (For instance, if the weekly work hours, h, of an employee must be at least 40 or $h \geq 40$, any value that is 40 or greater meets this constraint.) As with equations, a solution to an inequality is any value that makes the inequality true. All of these values taken together form the solution set to the inequality.

Lesson 18 occurs after administering the Unit 2 assessment and includes post-assessment activities. Taking this time to pause after the assessment to collect student reflection data through a survey and teacher conferences is a critical aspect of the course and building the classroom culture. The Student Survey is an opportunity to gather low-stakes, non-evaluative feedback for teachers to support equity and instructional pedagogy.

Instructional Routines

Aspects of Mathematical Modeling: Lessons 1, 9 &10, 12, 13 & 14, 15, 17

Card Sort: Lessons 5, 18

Co-Craft Questions (MLR5): Lessons 9 & 10, 12, 16

Collect and Display (MLR2): Lessons 4, 6, 11, 15

Compare and Connect (MLR7): Lessons 1, 3, 11, 17

Critique, Correct, Clarify (MLR3): Lesson 17

Discussion Supports (MLR8): Lessons 2, 5, 6, 7

Math Talk: Lessons 2, 6

Notice and Wonder: Lesson 5

Round Robin: Lessons 6, 8, 13 &14

Stronger and Clearer Each Time (MLR1): Lessons 2, 4

Take Turns: Lessons 5, 6, 7, 18

Three Reads (MLR6): Lessons 8, 12

Lesson 1: Planning a Pizza Party

PREPARATION

Lesson Goals	Learning Targets
• Comprehend the term "constraint" to mean a limitation on the possible or reasonable values a quantity could have. • Use variables and the symbols =, $<$, $>$, $\leq$, and $\geq$ to represent simple constraints in a situation. • Write expressions with numbers and variables to represent the quantities in a situation.	• I can explain the meaning of the term "constraints." • I can tell which quantities in a situation can vary and which ones cannot. • I can use numbers and variables to write expressions representing the quantities in a situation.

Lesson Narrative

This opening lesson invites students to experiment with expressions, equations, and inequalities to model a situation. Students think about relevant quantities, whether they might be fixed or variable, and how they might relate to one another. They make assumptions and estimates, and use numbers and variables to represent the quantities and relationships. The lesson also draws attention to the idea of **constraints** and how to represent them.

There is not one correct set of expressions, equations, or inequalities governing the potential quantities involved in the pizza party. The focus is on the modeling process itself: identifying relevant quantities, making assumptions, creating a **model**, and evaluating the model (MP4). Discussions are built in to foster an environment of collaboration and active thinking and listening. Encourage students to share their ideas and questions at these times.

In subsequent lessons, students will continue to write and interpret expressions, equations, and inequalities that represent situations and constraints.

Making internet-enabled devices available gives students an opportunity to choose appropriate tools strategically (MP5).

What do you hope to learn about your students during this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.6.EE.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem.	**NC.M1.A-SSE.1:** Interpret expressions that represent a quantity in terms of its context. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - Resources for researching pizza prices such as advertisements or access to the internet
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L1 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Addressing: NC.6.EE.6

The purpose of this bridge is for students to notice quantities and relationships in written situations, and connect them to operations on variables. This supports students in interpreting parts of expressions and equations as well as building their own expressions and equations later in this lesson and unit. This task is aligned to question 1 in Check Your Readiness.

Student Task Statement

A zookeeper is preparing to care for snakes in an exhibit. For every feeding, she needs three mice and an additional two mice for each snake. How many mice are needed if the number of the snakes is:

a. 10

b. 6

c. x

DO THE MATH	PLANNING NOTES

Warm-up: A Main Dish and Some Side Dishes (*5 minutes*)

Building On: NC.6.EE.6	**Addressing:** NC.M1.A-SSE.1	**Building Towards:** NC.M1.A-CED.1

This warm-up elicits the idea that an equation is a statement of equivalent expressions, and that different expressions represent a quantity. Students will interpret the expressions and equations in context. Later in this lesson and throughout the unit, students will create, interpret, and reason about equations with variables representing quantities.

Step 1

- Ask students to begin working on the warm-up independently for 1 minute, then to arrange themselves into pairs, or use visibly random grouping, and compare responses.

Student Task Statement

Here are some variables and what they represent. All costs are in dollars.

- m represents the cost of a main dish.
- n represents the number of side dishes.
- s represents the cost of a side dish.
- t represents the total cost of a meal.

Equation	What does the equation mean in this situation?
a. $m = 7.50$	
b. $m = s + 4.50$	
c. $ns = 6$	
d. $m + ns = t$	

Step 2

Facilitate a whole-class discussion for students to define what an equation is and share their interpretations of what the equations and expressions represent.

- Begin discussion with a focus on defining what an equation is. For each question, have students turn and talk with a partner before sharing with the full class.
  - "What is an equation? What does it tell us?" (An equation is a statement that an expression has the same value as another expression.)
  - "Can equations contain only numbers?" (Yes; $3 + 4.50 = 7.50$) "Only variables?" (Yes; $m + ns = t$) "A mix of numbers and variables?" (Yes; $m = s + 4.50$)
  - "What are variables used for in equations?" (They represent quantities. For example, *m* represented the cost of a main dish in dollars and *n* represented the number of side dishes.)
  - "One equation tells us that a main dish is \$7.50. Another equation tells us that it is equal to the expression $s + 4.50$. Could both be true? Are they both appropriate for expressing the cost of a main dish?" (Yes. The first one tells us the price in dollars. The second tells us how the price of a main dish compares to the price of a side dish. It is \$4.50 more.)
- Invite students to share their interpretations of the given equations. Some possible questions are:
  - “What does ns represent? How do you know?” (The expression ns is the cost of n sides. This is because the expression multiplies the number of sides n and the cost of a side s.)

– "What does $m + ns$ represent? How do you know?" (The expression m is the cost of a main dish. The expression ns is the cost of n sides. Adding the two expressions will give the total cost for a main dish and n sides.)

DO THE MATH	PLANNING NOTES

Activity 1: How Much Will It Cost? (*15 minutes*)

Instructional Routine: Aspects of Mathematical Modeling	
Addressing: NC.M1.A-SSE.1	**Building Towards:** NC.M1.A-CED.1; NC.M1.A-CED.2;

This activity prompts students to engage in *Aspects of Mathematical Modeling* by creating expressions to represent the quantities and relationships in a situation. The situation that is presented is planning a pizza party for the class; however, this situation can be modified to match an actual event at the school and/or select a different food choice. Personalizing the context for the students will help students connect to the activity.

Students plan a pizza party and present a cost estimate. To do so, they need to consider relevant variables, make assumptions and estimates, perform calculations, and adjust their thinking along the way (MP4). Some students may choose to perform research and revise their models as they gather new information. There are many possible solutions to the task.

Step 1

- Ask students if they have ever been in charge of planning a party. Solicit a few ideas of what party planners need to consider. Ask students to imagine being in charge of a class pizza party. Explain that their job is to present a plan and a cost estimate for the party.
- Ask students to arrange themselves in groups of four or use visibly random grouping. Provide access to calculators and, if feasible and desired, access to the internet for researching pizza prices. Students can also make estimates based on prior experience, refer to printed ads, or use their personal device to look up pricing information.
- Limit the time spent on the first question to 5–6 minutes and pause the class before students move on to subsequent questions.

Monitoring Tip: As students discuss their ideas, monitor for those who:
- find and use actual data or exact values (for example, count the number of students in the class, research the cost of a large pizza at a nearby shop, or quickly survey the class for topping preferences).
- estimate quantities based on prior knowledge (for example, the cost of a large pizza in a recent purchase, or the number of slices they and their friends generally consume at lunch time).
- make assumptions about behaviors, preferences, or quantities (for example, assume that a certain percentage of the class prefers a certain topping).

Step 2

- Give groups of students 1–2 minutes to share their proposals with another group. Then, select a few groups who used contrasting strategies (such as those outlined in the Monitoring Tip) to briefly share their plans with the class. Record or display their plans for all to see.
- Ask students to complete the remaining questions. If needed, give an example of an expression that can be written to represent a cost calculation.

While groups work, intentionally seek to encourage individuals who do not typically volunteer to be ready to share a specific contribution they made within their groups (elevate the status of contributions for students' who otherwise may be undervalued). Let them know you will be calling on them, and specifically what you want them to share, as you monitor.

Advancing Student Thinking: If students do not understand what is meant by "quantities that might change," ask them if it is more likely that the cost of a pepperoni pizza increases on the day of the party or that some students are absent that day. In a model that incorporates both of these quantities, they may wish to use a number for the cost of each type of pizza and a letter for the number of students present that day.

Student Task Statement

Imagine your class is having a pizza party.

Work with your group to plan what to order and to estimate what the party would cost.

1. Record your group's plan and cost estimate. What would it take to convince the class to go with your group's plan? Be prepared to explain your reasoning.
2. Write down one or more expressions that show how your group's cost estimate was calculated.
    a. In your expression(s), are there quantities that might change on the day of the party? Which ones?
    b. Rewrite your expression(s), replacing the quantities that might change with variables. Be sure to specify what the variables represent.

Are You Ready For More?

Find a pizza place near you and ask about the diameter and cost of at least two sizes of pizza. Compare the cost per square inch of the sizes.

Step 3

- Invite groups who did not previously share their plans to share the expressions they wrote and explain what the expressions represent. Expressions shared do not need to be the most sophisticated. Record the expressions for all to see.
- If student expressions are all numeric:
    – Record the quantities students mention and display them for all to see. For example, the number of students in the class, the number of pizza slices per person, the cost of delivery, the price per topping.
    – Briefly discuss the quantities that students anticipate would change and therefore replace with variables.
    – Rewrite the expressions to include the variables.
    – Then continue with the discussion on interpreting the parts of the expressions.

- If student expressions include variables:
  - Ask students to specify what their variables represent. (c represents the number of cheese pizzas; n represents the number of students.)
  - Ask students to interpret parts of their expressions. For example, if students wrote $4c + 3p$ to represent the total cost for pizza, what does the term $4c$ represent? (the cost for all of the cheese pizzas) If students wrote $(2n) \div 8$ to represent the number of pizzas, what does $2n$ represent? (the total number of slices if each person has two slices each)
- After each group shares, ask if others calculated the costs the same way but wrote different expressions.

DO THE MATH	PLANNING NOTES

Activity 2: What are the Constraints? (*10 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Addressing: NC.M1.A-CED.1	**Building Towards:** NC.M1.A-CED.2

Writing and solving equations and inequalities often revolves around the idea of representing and satisfying constraints. This activity introduces the term "constraints" and begins to develop the idea that expressions, equations, and inequalities can help us describe constraints on quantities. It prompts students to recognize that quantities are sometimes constrained in terms of the values they could take or in terms of how they relate to another quantity.

RESPONSIVE STRATEGIES

Display or provide charts with symbols and meanings. Invite students to name additional examples of other variables that might be considered constraints to encourage critical thinking and application to the expressions created in this activity. Examples of additional constraints might include t < 2 to represent how long the party could last, or b ≤ 2 to say that each student will get no more than 2 beverages.

Supports accessibility for: Conceptual processing; Memory

Step 1

- Tell students that they will now look at some constraints of the pizza party. Explain that a constraint is something that limits what is possible or what is reasonable in a situation. For example, one constraint a teacher has to work with is the amount of time in a class period or the number of school days in a year (both of these might be a fixed number). Another constraint might be the number of students in a class (which may vary by class, but is usually no more than a certain number).
- Keep students in groups of four. Give students 2 minutes of quiet work time and then time to briefly share their responses with their group.

Advancing Student Thinking: If students have trouble thinking of constraints for a chosen variable, ask about extreme values. For instance, ask: "Do you think a large pizza might cost $100? How about $3?"

Some students may struggle with translating the written descriptions of the constraints into inequalities. For example, “The greatest number of students in the class would be 30” might be mistakenly written as $s \geq 30$. Ask these students to explain the meaning of the “$\geq$” symbol. Ask: "If 29 students come to class, can we write $29 \geq 30$? Can we write $30 \geq 30$?"

Student Task Statement

A constraint is something that limits what is possible or reasonable in a situation.

For example, one constraint in a pizza party might be the number of slices of pizza each person could have, s. We can write $s \leq 3$ to say that each person gets three or fewer slices.

1. Look at the expressions you wrote when planning the pizza party earlier.
    a. Choose an expression that uses one or more variables.
    b. For each letter, determine what values would be reasonable. (For example, could the value be a non-whole number? A number greater than 50? A negative number? Exactly 2?)
2. Write equations or inequalities that represent some constraints in your pizza party plan. If a quantity must be an exact value, use the $=$ symbol. If it must be greater or less than a certain value to be reasonable, use the $<$, $>$, $\leq$ or $\geq$ symbol.

Step 2

Use the *Compare and Connect* routine to invite students to examine several different equations or inequalities that represent the constraints in their party plan. For example, ask students to identify one or more of the following:

- A constraint that is an exact value. For example, ordering pizza online involves a fee of $2.50 ($f = 2.50$), or the price of a large cheese pizza is $9 ($p = 9$).
- A constraint that involves a boundary or a limit. For example, an order must be at least $25 in value to qualify for free delivery ($t \geq 25$), or the party must cost no more than $90 $(c \leq 90)$.
- An inequality that states both a lower and an upper limit. For example, the number of students would be between 0 and 30 (assuming 30 is the number of students in the class) $(0 \leq s \leq 30)$.

As students identify these different types of constraints, ask them to make comparisons and connections across the ways the class has represented their different constraints (for example, how two students represented fixed costs or how two students represented two different upper limits in the same way, using an inequality). Listen for and amplify the language they are using and how the symbols in the equations and inequalities communicate the meaning of the values.

Why This Routine? Use *Compare and Connect* (MLR7) to foster students’ meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students used expressions, equations, and inequalities to model constraints in a real-life situation. They also identified quantities that might change and interpreted expressions and parts of expressions in terms of the context. The goal of the debrief is to discuss the process of creating models, specifically the use of expressions, equations, and inequalities to represent constraints.

Tell students that the expressions, equations, and inequalities they wrote are mathematical models. A model is a mathematical representation of a real-life situation. When people create models, they rely on the information they have, but they also make assumptions and decisions that affect the models. If the information or assumptions change, the model would also change.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "In planning a pizza party, what were some ways we gathered information to estimate the cost?" (counting the number of students, researching actual prices)
- "What were some assumptions we made?" (pizza preferences, number of slices per person, possible prices of pizza)
- "Suppose we had gathered information differently, for example, by asking every student the exact pizza toppings and number of slices. Would that have been a reasonable approach? How would that have changed the cost estimate? " (It would be inefficient to take exact orders, cost much more, and likely mean a lot of leftovers.)
- "Suppose we had made a different set of assumptions, for example, assuming that everyone loved pepperoni and would like only one slice. How would that have changed the cost estimate?" (If the assumption was a slice of pepperoni pizza per person, it would probably cost less, but many students might not be able to enjoy the pizza or might not have enough.)
- "In planning the party, we saw some examples of constraints. Can you think of some constraints in other situations? What might be some constraints in, say, planning a field trip, or in organizing a community service event?" (A constraint for a field trip could be the number of buses or chaperones available. A community service event could be constrained by the available budget or space to hold the event.)

PLANNING NOTES

Student Lesson Summary and Glossary

The expressions, equations, and inequalities you wrote in this lesson are mathematical **models**.

> **Model**: A mathematical representation used to describe quantities and their relationships in a real-life situation. It can be used to solve problems and make decisions.

Often, what we want to describe are **constraints**.

> **Constraint**: Something that limits what is possible or what is reasonable in a situation.

For example, when planning a birthday party, we might be dealing with these quantities and constraints:

quantities
- the number of guests
- the cost of food and drinks
- the cost of birthday cake
- the cost of entertainment
- the total cost

constraints
- 20 people maximum
- $5.50 per person
- $40 for a large cake
- $15 for music and $27 for games
- no more than $180 total cost

We can use both numbers and variables to represent the quantities. For instance, we can write 42 to represent the cost of entertainment, but we might use the letter n to represent the number of people at the party and the letter C for the total cost in dollars.

We can also write expressions using these numbers and variables. For instance, the expression $5.50n$ is a concise way to express the overall cost of food if it costs $5.50 per guest and there are n guests.

Sometimes a constraint is an exact value. For instance, the cost of music is $15. Other times, a constraint is a boundary or a limit. For instance, the total cost must be no more than $180. Symbols such as <, >,≤,≥ and = can help us express these constraints.

quantities
- the number of guests
- the cost of food and drinks
- the cost of birthday cake
- the cost of entertainment
- the total cost

constraints
- $n \leq 20$
- $5.50n$
- 40
- $15 + 27$
- $C \leq 180$

Equations can show the relationship between different quantities and constraints. For example, the total cost of the party is the sum of the costs of food, cake, entertainment. We can represent this relationship with:

$$C = 5.50n + 40 + 15 + 27 \qquad or \qquad C = 5.50n + 82$$

Deciding how to use numbers and variables to represent quantities, relationships, and constraints is an important part of mathematical modeling. Making assumptions—about the cost of food per person, for example—is also important in modeling.

A model such as $C = 5.50n + 82$ can be an efficient way to make estimates or predictions. When a quantity or a constraint changes, or when we want to know something else, we can adjust the model and perform a simple calculation, instead of repeating a series of calculations.

Cool-down: Ice Cream Party (*5 minutes*)

Addressing: NC.M1.A-SSE.1; NC.M1.A-CED.1

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

RESPONSIVE STRATEGY
Provide students with a list of constraints such as the number of students in their grade-level. A list of items that would normally be needed for a party, and estimated costs for specific items.

Cool-down

As a reward for achieving their goals, all students in your grade level are invited to an ice cream party.

1. Write an expression that could represent an estimated cost for the party. Use at least one letter. State what each part of the expression represents.
2. Choose a letter in your expression. Describe the values that would be reasonable for the quantity that the letter represents.

Student Reflection:

How do you feel about your level of input during class today (verbal or non-verbal)? What is something you'd like to do differently in the future?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Identify the ways in which the math community you are working to foster are going well. What aspects would you like to work on? What actions can you take to improve those areas?

Practice Problems

1. A balloon is released from a height of 5 feet and increases in height by 6.3 feet per second. Which expression represents the height of the balloon after x seconds?

    a. $5(6.3x)$
    b. $5+6.3x$
    c. $\frac{6.3x}{5}$
    d. $11.3x$

2. To support a local senior citizens center, a student club sent a flyer home to the n students in the school. The flyer said, "Please bring in money to support the senior citizens center. Paper money and coins accepted!" Their goal is to raise T dollars.

    Match each quantity to an expression, an equation, or an inequality that describes it.

Quantity	Expression, equation, or inequality
a. the dollar amount the club would have if they reached half of their goal	1. $T+50$
b. the dollar amount the club would have if every student at the school donated 50 cents to the cause	2. $0.5T$
c. the dollar amount the club could donate if they made \$50 more than their goal	3. $0.25n$
d. the dollar amount the club would still need to raise to reach its goal after every student at the school donated 50 cents	4. $0.5n$
e. the dollar amount the club would have if half of the students at the school each gave 50 cents	5. $T-0.5n$

3. A student has scored a 76, 82, 80, 95 on the first four quizzes in math class. Write an expression that represents the average quiz score if the student scores x on the fifth quiz.

4. A student club started a fundraising effort to support animal rescue organizations. The club sent an information flyer home to the n students in the school. It says, "We welcome donations of any amount, including any change you could spare!" Their goal is to raise T dollars, and to donate to a cat shelter and a dog shelter.

    Match each quantity to an expression, an equation, or an inequality that describes it.

Quantity	Expression, equation, or inequality
a. the dollar amount the club would still need to raise to reach its goal after every student at the school donated a quarter	1. $\frac{3}{4}n \cdot \frac{1}{2}$
b. the dollar amount the club would have if every student at the school donated a quarter to the cause	2. $\frac{1}{4}T$
c. the dollar amount the club would have if three-fourths of the students at the school each gave 50 cents	3. $T-\frac{1}{4}n$
d. the dollar amount the club could donate to the cat shelter if they reached their goal and gave a quarter of the total donation to a dog shelter	4. $\frac{3}{4}T$
e. the dollar amount the club would have if they reached one-fourth of their goal	5. $\frac{1}{4}n$

5. A softball team is ordering pizza to eat after their tournament. They plan to order cheese pizzas that cost $6 each and four-topping pizzas that cost $10 each. They order c cheese pizzas and f four-topping pizzas. Which expression represents the total cost of all of the pizzas they order?

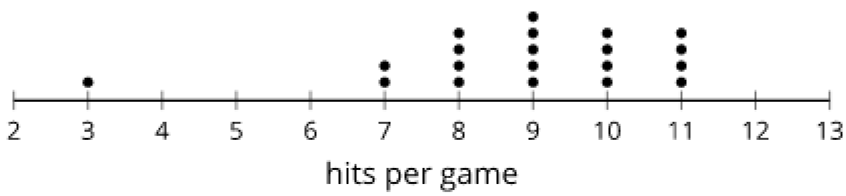
   a. $6 + 10$
   b. $c + f$
   c. $6c + 10f$
   d. $6f + 10c$

6. The values of coins in the pockets of several students are recorded. Find the mean of the values: 10, 20, 35, 35, 35, 40, 45, 45, 50, 60

   a. 10 cents
   b. 35 cents
   c. 37.5 cents
   d. 50 cents

   (From Unit 1)

7. The dot plot displays the number of hits a baseball team made in several games. The distribution is skewed to the left.



   If the game with 3 hits is considered to be recorded in error, it might be removed from the data set. If that happens:

   a. What happens to the mean of the data set?

   b. What happens to the median of the data set?

   (From Unit 1)

8. A set of data has a standard deviation of 0, and one of the data values is 14. What can you say about the data values?

   (From Unit 1)

9. A zookeeper is preparing to care for snakes in an exhibit. For each question, write an expression representing the supplies needed.

   a. The zookeeper needs 4.5 ounces of crickets for each snake. How many ounces of crickets are needed if the number of snakes is:

      - 10
      - 6
      - x

   b. There is one male snake, and the rest are female. The zookeeper needs one vitamin pill for every female snake. How many vitamin pills does she need if the number of snakes is:

      - 10
      - 6
      - x

   (Addressing NC.6.EE.6)

Lesson 2: Writing Equations to Model Relationships (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Given a description of a situation or an equation, identify quantities that vary and quantities that don't. • Understand that variables can be used to represent both quantities that vary and those that are constant. • Write equations with numbers and variables to describe relationships and constraints.	• I can tell which quantities in a situation can vary and which ones cannot. • I can use numbers and variables to write equations representing the relationships in a situation.

Lesson Narrative

This is the first of two lessons where students write equations to model various situations. The work here progresses in two ways—in terms of the complexity of the relationships and in terms of the amount of scaffolding built into the prompts.

Students begin by revisiting ways to calculate a given percentage of a given number, in preparation for computations they'll need to do in the lesson. Next, they look at a couple of contexts on spending, earning, and sales tax, which involve multiplication, multiplication and addition, and increasing a number by a percentage.

In each case, students begin by creating models where the values of the quantities are known (or mostly known), and move toward models where the quantities are unknown or can change. The repeated reasoning allows students to practice looking for and expressing regularity (MP8). As they interpret verbal descriptions and write equations, students develop their understanding of equations as a way to represent constraints and practice reasoning quantitatively and abstractly (MP2).

Which Standards for Mathematical Practice do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Addressing
NC.6.EE.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. **NC.6.RP.4:** Use ratio reasoning to solve real-world and mathematical problems with percents by: ● Understanding and finding a percent of a quantity as a ratio per 100. ● Using equivalent ratios, such as benchmark percents (50%, 25%, 10%, 5%, 1%), to determine a part of any given quantity. ● Finding the whole, given a part and the percent.	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L2 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.6	**Building Towards:** NC.M1.A-CED.1

The goal of this bridge is to help students practice identifying and representing pertinent information in a word problem. Later in this lesson, students create equations that represent relationships between two or more variables in contexts. This activity prepares students to be successful by helping them interpret word problems and determine if a variable is needed or not before they write equations to represent the relationships. This task is aligned to question 1 in Check Your Readiness.

Student Task Statement

For each problem, identify any important quantities. If it's a known quantity, write the number and a short description of what it represents. If it's an unknown quantity, assign a variable to represent it and write a short description of what that variable represents.

1. Elena is going to mow lawns for the summer to make some extra money. She will charge $20 for every lawn she mows and plans on mowing several lawns each week.

2. Tyler is packing his bags for vacation. He plans to pack two outfits for each day of vacation.

DO THE MATH	PLANNING NOTES

Warm-up: Percent of 200 (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.6.RP.4	

This *Math Talk* invites students to use what they know about fractions, decimals, and the meaning of percent to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students calculate prices that involve a percent increase and write an equation to generalize the calculation.

Why This Routine? A *Math Talk* builds fluency by encouraging students to think about the numbers, shapes, or algebraic expressions and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. While participating in these activities, there is a natural need for students to be precise in their word choice and use of language (MP6). Additionally, a Math Talk often provides opportunities to notice and make use of structure (MP7).

Finding different percents of the same value (200) is also an opportunity to reason repeatedly and look for and make use of structure (MP7, MP8).

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Step 1

- Display one problem at a time.
- Give students quiet think time for each problem.
- Ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.

Student Task Statement

Evaluate mentally.

- 25% of 200
- 12% of 200
- 8% of 200
- p % of 200

Step 2

- Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate ___'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ___'s strategy?"
  - "Do you agree or disagree? Why?"

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because...." or "I noticed ____ so I...." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

If one of the following strategies is not mentioned, consider sharing it with students:

- Convert each percentage into a fraction and multiply the fraction by 200. For example, they may think of 25% as $\frac{1}{4}$, 12% as $\frac{12}{100}$ or $\frac{3}{25}$, and 8% as $\frac{8}{100}$ or $\frac{2}{25}$.
- Convert each percentage into a decimal and multiply it by 200.
- Notice that 1% of 200 is 2, and that any percentage of 200 can be found by multiplying the percentage by 2. For example, 25% of 200 is $25 \cdot 2$, and $p\%$ of 200 is $p \cdot 2$ or $2p$.

DO THE MATH	PLANNING NOTES

Activity 1: Blueberries and Earnings (*15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)
Addressing: NC.M1.A-CED.2

In this activity, students write equations to represent quantities and relationships in two situations. In each situation, students express the same relationship multiple times: initially using numbers and variables, and later using only variables. The progression helps students see that quantities can be known or unknown, and can stay the same or vary, but both kinds of quantities can be expressed with numbers or variables.

Step 1

- Students work individually on problems 1–3. Let them know their responses to problem 3 will be first drafts, so initial ideas are fine.

Monitoring Tip: Monitor student strategies. Look for and highlight the strategy of initially using numbers to explore the relationship between the quantities. This can help in recognizing how the quantities are related. It also helps in identifying which quantities are unknown or can change versus the quantities that are constant. Remind students that variables are often used to represent those unknown or changing quantities.

Step 2

- Use the *Stronger and Clearer Each Time* routine for students to revise their initial thinking for problem 3. Pair students up to share their ideas or first draft with a partner and give/receive feedback in the form of clarifying questions and additional ideas. After two rounds of pair conversation, ask students to revise their first draft response into a second draft that is stronger and clearer than their initial thinking. Encourage students to incorporate good ideas they got from their partners into their second draft.

RESPONSIVE STRATEGIES

Leverage choice around perceived challenge. Invite students to write equations for 1-2 of the situations they select from problems 1 and 2. Chunking this task into more manageable parts may also benefit students who benefit from additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

***Advancing Student Thinking*:** Students may translate “Mai earned m dollars, which is 45 more dollars than Noah did” as $m+45=v$, not paying attention to where the plus sign should go. As with other problems throughout this unit, encourage students to try using numbers in their equation to see if the equation really says what they want it to say.

Student Task Statement

1. Three friends visit a local Farmer’s Market and purchase blueberries. Write an equation to represent each situation.
   a. Blueberries are \$4.99 per pound. Paul buys $\frac{3}{4}$ pound of blueberries and pays r dollars.
   b. Blueberries are \$4.99 per pound. Diego buys b pounds of blueberries and pays \$14.95.
   c. Blueberries are \$4.99 per pound. Jada buys p pounds of blueberries and pays c dollars.
   d. Blueberries are d dollars per pound. Lin buys q pounds of blueberries and pays t dollars.
2. Noah and Mai have summer jobs. Write an equation to represent each situation.
   a. Noah earned \$400 over the summer. Mai earned p dollars, which is \$45 more than Noah did.
   b. Noah earned n dollars over the summer. Mai earned \$275, which is \$45 more than Noah did.
   c. Noah earned v dollars over the summer. Mai earned m dollars, which is 45 dollars more than Noah did.
   d. Noah earned w dollars over the summer. Mai earned x dollars, which is y dollars more than Noah did.
3. How are the equations you wrote for the blueberry purchases like the equations you wrote for Mai and Noah’s summer earnings? How are they different?

Step 3

Facilitate a whole-class discussion.

- Sequence and connect a few students’ second drafts to discuss with the class. Many students will notice that the blueberry equations involve multiplication (or division) and the earnings equations involve addition (or subtraction)
- Call attention to second drafts that mention that some quantities are known or are fixed and others are not, ask them to specify which ones are which.

DO THE MATH	PLANNING NOTES

Activity 2: Car Prices (*10 minutes*)

Addressing: NC.M1.A-CED.2

This activity gives students another opportunity to represent a relationship with numbers and variables, to reason repeatedly, and to see more clearly that equations are helpful for generalizing a relationship.

From the given descriptions, students are aware that there are four relevant quantities in the car purchase situations. In each situation, the value of at least one quantity is not given, creating a need for students to name it, either using a word or a phrase (for instance, "total price" or "original price") or to use a variable (for example, T or p). Some students might choose to use a symbol.

Step 1

- Facilitate a discussion on the costs and fees associated with purchasing a car, asking students to turn and talk to a partner before sharing with the full class.
  - Ask, “If a car is listed at a dealership for $9500, can you show up with exactly $9500 and be good to buy the car? Why or why not?” Use student responses to generate a list of additional costs such as sales tax and fees.
- Ask students if they have had to pay sales tax when making a purchase and, if so, to briefly explain how sales tax works.
  - Explain to students that a car purchase also involves a sales tax. Car buyers pay not only the price of a car, but also a tax that is a certain percentage of the car price. Car dealerships also often charge their customers various fees.
- Tell students that they will now write equations to describe the relationship between the price of the car, the tax, a fee, and the total price. Emphasize that it is not necessary to evaluate any expressions or perform any computations.
- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 2 minutes of quiet work time and then 3 minutes to discuss their responses with their partner.

Monitoring Tip: Monitor for different ways students represent the relationship with equations. Students may:
- include only numerical values such as $9500 + 0.03(9500) + 120 = 9905$
- include a letter to represent the total cost in dollars such as $T = 9500 + 0.03(9500) + 120$
- combine like terms such as $T = 1.03(9500) + 120$

Identify students writing equivalent equations and ask them to share during discussion later.

Advancing Student Thinking: Some students may be taken aback by the prompt to write an expression relating four quantities. If they have trouble getting started, suggest that they simply calculate the cost of buying a $9,500 car, taking care to show their work.

One part of the question gives the total price of purchase rather than the original price of the car. If students use the given value as an original price, ask them to double-check the given information. If students get stuck, ask them how to consider where in the equation the total price is represented in parts a and b.

Student Task Statement

The tax on the sale of a car in North Carolina is 3%. At a dealership in Concord, a car purchase also involves \$120 in miscellaneous charges.

There are several quantities in this situation: the original car price, sales tax, miscellaneous charges, and total price. Write an equation to describe the relationship between all the quantities when:

a. The original car price is \$9,500.

b. The original car price is \$14,699.

c. The total price is \$22,480.

d. The original price is p.

Are You Ready For More?

How would each equation you wrote change if the tax on car sales is r% and the miscellaneous charges are m dollars?

Step 2

- Select students whose equations are equivalent but in different forms to share their responses. Record and display them for all to see.
- Focus on equations written for parts a and d in the student task statement. Those equations might be: $T = 9,500 + 0.03(9,500) + 120$ and $T = p + 0.03p + 120$. Ask students:
    - "In the first equation, what quantities do we know?" (the original price of the car, the tax rate, and the miscellaneous charges)
    - "When might it be useful to write an equation like this?" (when we know all relevant quantities except for one)
    - "In the other equation, what quantities do we know?" (the tax rate and the miscellaneous charges)
    - "When might it be useful to write an equation like this?" (when we want to have a kind of formula for finding the total price for a car of any price, assuming the tax rate and miscellaneous charges are fixed at 3% and \$120 respectively)
- Emphasize that we might choose to use variables to represent quantities that vary or those that are constant, depending on what we want to understand or know.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students wrote expressions and equations to describe scenarios, using variables to represent quantities that could change or are unknown. The goal of this debrief is to have students think about why—or why not—we might want to use variables in an expression or equation. Facilitate a discussion using the following questions.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. • We could use numbers or variables to represent the quantities in a situation. When might it make sense to use only numbers? When might it make sense to use variables? • You've heard the phrases "a quantity that varies" and "a quantity that stays constant" in this lesson. Describe what they mean in your own words. If possible, give an example of a situation that has a quantity that varies and a quantity that stays constant.	**PLANNING NOTES**

Student Lesson Summary and Glossary

Suppose your class is planning a trip to a museum. The cost of admission is \$7 per person and the cost of renting a bus for the day is \$180.

- If 24 students and 3 teachers are going, we know the cost will be: $7(24)+7(3)+180$ or $7(24+3)+180$.
- If 30 students and 4 teachers are going, the cost will be: $7(30+4)+180$.

Notice that the numbers of students and teachers can vary. This means the cost of admission and the total cost of the trip can also vary, because they depend on how many people are going.

Variables are helpful for representing quantities that vary. If s represents the number of students who are going, t represents the number of teachers, and C represents the total cost, we can model the quantities and constraints by writing:

$$C = 7(s+t)+180$$

Some quantities may be fixed. In this example, the bus rental costs \$180 regardless of how many students and teachers are going (assuming only one bus is needed).

Variables can also be used to represent quantities that are constant. We might do this when we don't know what the value is, or when we want to understand the relationship between quantities (rather than the specific values).

For instance, if the bus rental is B dollars, we can express the total cost of the trip as $C = 7(s+t)+B$. No matter how many teachers or students are going on the trip, B dollars need to be added to the cost of admission.

Cool-down: Shirt Colors (*5 minutes*)

Building Towards: NC.M1.A-CED.2

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding. Lessons 4 and 8 provide additional opportunities to analyze the meaning of variables in equations and write equations to represent a situation.

Cool-down

A school choir needs to make T-shirts for its 75 members and has set aside some money in its budget to pay for them. The members of the choir decided to order from a printing company that charges \$3 per shirt, plus a \$50 fee for each color to be printed on the shirts.

1. Write an equation that represents the relationship between the number of T-shirts ordered, the number of colors on the shirts, and the total cost of the order. If you use any variables, specify what they represent.

2. In this situation, which quantities do you think can vary? Which might be fixed?

Student Reflection:

What resources or practices in math class help you learn best?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

As students wrote equations today, what evidence did you see that they are looking for and expressing repeated reasoning (MP8)?

Practice Problems

1. Large cheese pizzas cost $5 each, and large one-topping pizzas cost $6 each.

   Write an equation that represents the total cost, T, of c large cheese pizzas and d large one-topping pizzas.

2. Jada plans to serve milk and healthy cookies for a book club meeting. She is preparing 12 ounces of milk and 4 cookies per person. Including herself, there are 15 people in the club. A package of cookies contains 24 cookies and costs $4.50.

   A 1-gallon jug of milk contains 128 ounces and costs $3. Let n represent number of people in the club, m represent the ounces of milk, c represent the number of cookies, and b represent Jada's budget in dollars.

   Select **all** of the equations that could represent the quantities and constraints in this situation.

   a. $m = 12(15)$
   b. $3m + 4.5c = b$
   c. $4n = c$
   d. $4(4.50) = c$
   e. $b = 2(3) + 3(4.50)$

3. A student on the track team runs 45 minutes each day as a part of her training. She begins her workout by running at a constant rate of 8 miles per hour for a minutes, then slows to a constant rate of 7.5 miles per hour for b minutes.

   Which equation describes the relationship between the distance she runs in miles, D, and her running speed, in miles per hour?

   a. $a + b = 45$
   b. $8a + 7.5b = D$
   c. $8(\frac{a}{60}) + 7.5(\frac{b}{60}) = D$
   d. $8(45 - b) + 7.5b = D$

4. Elena bikes 20 minutes each day for exercise.

   Write an equation to describe the relationship between her distance in miles, D, and her biking speed, in miles per hour, when she bikes:

   a. at a constant speed of 13 miles per hour for the entire 20 minutes

   b. at a constant speed of 15 miles per hour for the first 5 minutes, then at 12 miles per hour for the last 15 minutes

   c. at a constant speed of M miles per hour for the first 5 minutes, then at N miles per hour for the last 15 minutes

5. A coach for a little league baseball team is ordering trophies for the team. Players on the team are allowed to choose between two types of trophies. The gold baseball trophies cost $5.99 each, and the uniform baseball trophies cost $6.49 each. The team orders g gold baseball trophies and u uniform baseball trophies.

   Write an expression that could represent the total cost of all of the trophies.

   (From Unit 2, Lesson 1)

6. In a trivia contest, players form teams and work together to earn as many points as possible for their team. Each team can have between three and five players. Each player can score up to 10 points in each round of the game. Elena and four of her friends decided to form a team and play a round.

   Write an expression, an equation, or an inequality for each quantity described here. If you use a variable, specify what it represents.

   a. the number of points that Elena's team earns in one round

   b. the number of points Elena's team earns in one round if every player scores between six and eight points

   c. the number of points Elena's team earns if each player misses one point

(*continued*)

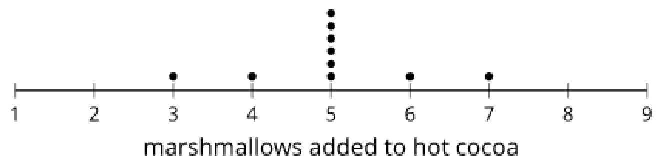
d. the number of players in a game if there are five teams of four players each

e. the number of players in a game if there are at least three teams

(From Unit 2, Lesson 1)

7. The dot plot displays the number of marshmallows added to hot cocoa by several kids. Which is the standard deviation of the data represented in the dot plot?

a. 0.2 marshmallow
b. 1 marshmallow
c. 2 marshmallows
d. 5 marshmallows



(From Unit 1)

8. Here is a data set from a research study:

5	10	10	15	100

a. After studying the data, the researcher realized that the value 100 was meant to be recorded as 15. What happens to the mean and standard deviation of the data set when the 100 is changed to a 15?

b. For the original data set, with the 100, would the median or the mean be a better choice of measure for the center? Explain your reasoning.

(From Unit 1)

9. For each problem, identify any important quantities. If it's a known quantity, write the number and a short description of what it represents. If it's an unknown quantity, assign a variable to represent it and write a short description of what that variable represents.

a. Clare is in charge of getting snacks for a road trip with her friends and her dog. She has $35 to go to the store to get some supplies. The snacks for herself and her friends cost $3.25 each, and her dog's snacks cost $9 each.

b. Mai's teacher orders tickets to the local carnival for herself, the entire class, and three more chaperones. Student tickets are $4.50.

(Addressing NC.6.EE.6)

10. Twenty-five people were attending an event. The ages of the people are as follows:[1]

3, 3, 4, 4, 4, 4, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 16, 17, 22, 22, 25.

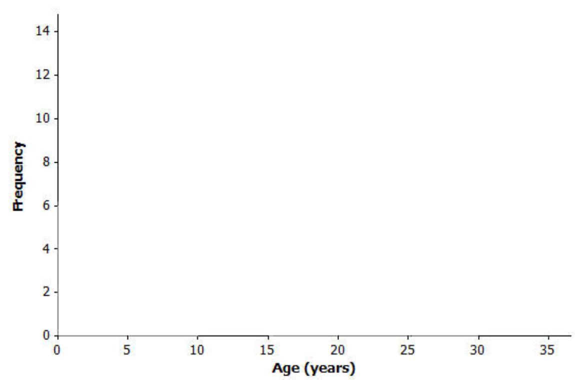
a. Create a histogram of the ages using the provided axes.

b. Would you describe your graph as symmetrical or skewed? Explain your choice.

c. Identify a typical age of the 25 people.

d. What event do you think the 25 people were attending? Use your histogram to justify your conjecture.

(From Unit 1)



[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Lesson 3: Writing Equations to Model Relationships (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Identify and describe (orally and in writing) patterns in tables of values and in calculations. • Use observed patterns to generalize the relationships between quantities and to write equations.	• I can use words and equations to describe the patterns I see in a table of values or in a set of calculations. • When given a description of a situation, I can use representations like diagrams and tables to help make sense of the situation and write equations for it.

Lesson Narrative

In this lesson, students continue to develop their ability to identify, describe, and model relationships with mathematics.

Previously, students worked mostly with descriptions of familiar relationships and were guided to reason repeatedly, which enabled them to see a general relationship between two quantities. Here, students are given tables of values and asked to generalize the relationship between pairs of quantities—by studying the values and looking for patterns (MP8), and by interpreting them in context (MP2). Some of the relationships they encounter here are novel or otherwise require perseverance and careful reasoning to pin down (MP1).

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.9: Represent and analyze quantitative relationships by: • Using variables to represent two quantities in a real-world or mathematical context that change in relationship to one another. • Analyze the relationship between quantities in different representations (context, equations, tables, and graphs).	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L3 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Addressing: NC.6.EE.9

The purpose of this bridge is for students to recall the connection between tables and equations. Later in this lesson, students are given tables and are asked to come up with an equation to represent a possible relationship describing all the sets of two associated numbers in the table. Here, they do the reverse. Students practice using an equation to complete a table by substituting one value and determining the other. In the process, students also practice solving equations. This task best aligns to question 5 in Check Your Readiness as students solve operations with rational numbers. Encourage students to share their reasoning.

Student Task Statement

Complete the table so that each pair of numbers makes the equation true.

$y = 3x$

x	y
-5	
	96
$\frac{2}{3}$	

DO THE MATH

PLANNING NOTES

Warm-up: Finding a Relationship (*10 minutes*)

Building Towards: NC.M1.A-CED.2

The activities in this lesson require students to observe tables of values, look for patterns, and generalize their observations into equations. This warm-up prompts students to think about how they could go about analyzing the values in the table and to articulate their reasoning. The relationship is one students may be less familiar with, which should provide opportunity to draw out different strategies.

Step 1

- Display the table for all to see. Explain that the quantities in each column are related. Emphasize that the goal is *not* to successfully find a relationship. It is to notice the strategies they use when attempting to figure out what the relationship might be.
- Give students 2 minutes to attempt at least one strategy independently, then ask students to arrange themselves into pairs, or use visibly random grouping, to share individual attempts and brainstorm additional strategies together.

Student Task Statement

Here is a table of values where the two quantities x and y are related.

x	y
1	0
3	8
5	24
7	48

What are some strategies you could use to find a relationship between x and y? Brainstorm as many ways as possible.

Step 2

- Invite groups to share their strategies and record them for all to see. If not already described by students, apply each strategy using the values in the table, or ask students to give an example of how it could be applied. Possible strategies might include:
  - Identifying an operation that could be done to x that would produce y. (The y-values are 1 less than the square numbers 1, 4, 9, 25, and 49. These numbers are the squares of the listed x-values. The sequence of operations could be expressed as, "square x and subtract 1 to get y" or by the equation $y = x^2 - 1$.)
  - Comparing how the x-values and y-values change to see if there is a pattern. (Each time x increases by 2, y increases by 8 more than the previous time.)
  - Seeing if the numbers in one column follow a special pattern and if that pattern could be connected to the numbers in the other column.

  Some students may plot the x and y pairs on a coordinate plane to check if the graph looks like a familiar relationship. Discuss this strategy with the class only if students bring it up. Graphing will not be introduced as a strategy in this course until Unit 3.
- Ask students to keep in mind the different strategies as they work on the activities in the lesson.

DO THE MATH

PLANNING NOTES

Activity 1: What Are the Relationships? (*20 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Addressing: NC.M1.A-CED.2	

In the previous lesson, students wrote equations to model relationships presented via verbal descriptions. This activity offers new opportunities to identify and represent relationships between pairs of quantities. In the first situation, students are presented with tables of values where students may reason with the context and consider operations to connect the two quantities. In the second situation, they are given a visual pattern that changes with each successive figure. Here students may reason with the diagram to determine a relationship. They may also decide to create a table of values to explore patterns. Lastly, students are presented with a proportional situation that involves three quantities. Students will need to reason carefully about how two of the quantities are related.

In each case, students need to interpret the values in context, look for structure or patterns, and generalize them (MP7). As they do so, students also practice reasoning quantitatively and abstractly (MP2).

RESPONSIVE STRATEGIES

Leverage choice around perceived challenge. Invite students to select 2 of the 3 tasks to complete. Chunking this task into more manageable parts may also benefit students who benefit from additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Step 1

- Keep students in pairs.
- Students work through the activity for 7–10 minutes.
- As students finish, invite them to create a visual display of one of the relationships, showing how the quantities are visible in different ways in each representation. This will set up a whole-class *Compare and Connect* routine.

Monitoring Tip: As students discuss their thinking, listen for the different ways they describe the same relationship. For example, here are some ways students might describe the relationship in the first table ("Meters from home" and "Meters from school"):

- The distance from home and the distance from school always add up to 400.
- The distance from school is always 400 minus the distance from home.
- As the distance from home, x, increases by a number, the distance from school, y, decreases by the same number. x starts at 0 and y starts at 400.
- The distance between home and school is 400 meters. The table seems to be telling us about a person traveling from home to school and how their distance to home and distance to school change along the way.

Select students with different descriptions to share later. Focus on descriptions that would generate different equations. These might be $x + y = 400$ versus $y = 400 - x$.

***Advancing Student Thinking*:** Encourage students who get stuck on question 1 to select one of the strategies discussed during the warm-up. Ask students “Is there an operation that could be done to x to produce y?” or “Compare how the x-values change and the y-values change. Is there a pattern?”

For students who get stuck on question 2, encourage them to figure out a way to count the number of hexagons without counting them one at a time. Ask students “How might you determine that there are 10 hexagons in figure 2 without counting 1, 2, 3, …, 10? Will that method work for the other figures too?”

Some students may have trouble getting started on the question about the volume of milk or setting up a table. Ask students which units are given in the problem, or suggest the headings “gallons,” “cups,” and “fluid ounces.” Then, ask them to use the given information to complete a row in the table. This might involve trying a different unit to start with. (For example, if they start with $\frac{1}{2}$ gallon and struggle to find the equivalent amount in cups and fluid ounces, try starting with 4 cups or 8 cups.) A more direct hint is to suggest finding the number of fluid ounces in 8 cups.

Student Task Statement

1. Each table below represents the relationship between two quantities.

Table A

Meters from home, x	0	75	128	319	396
Meters from school, y	400	325	272	81	4

Table B

Electricity bills in dollars, x	85	124	309	816
Total expenses in dollars, y	485	524	709	1,216

a. Describe in words how the two quantities in each table are related.

b. Write an equation to represent the relationship between the two quantities.

2. Each figure below is created using hexagons.

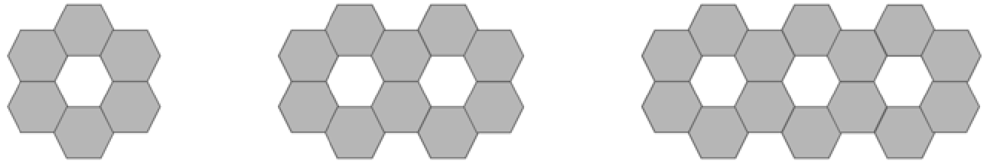


figure 1 figure 2 figure 3

a. What is the relationship between the figure number and the number of hexagons in the figure?

b. Write an equation to represent the relationship between the figure number and the number of hexagons.

c. Use your equation to determine the number of hexagons in figure 100.

3. A $\frac{1}{2}$-gallon jug of milk can fill 8 cups, while 32 fluid ounces of milk can fill 4 cups.

a. What is the relationship between the number of gallons and ounces?

b. Write an equation to represent the relationship between the two quantities.

Are You Ready For More?

Each figure to the right is created using stars.

What is the relationship between the figure number and the number of stars in the figure? Represent the relationship in as many ways as possible.

figure 1 *figure 2* *figure 3* *figure 4*

Step 2

- Identify which relationship elicited the most variety of reasoning and/or visual representation.
  - Use the *Compare and Connect* routine to invite students to make connections between the various ways of representing the relationship. Encourage students to clarify where and how they "see the quantities" in the relationship in the tables, diagrams, and/or equations, and to check any equations that look different for accuracy.
- Prompt students to make specific connections using questions and noticings such as the following:
  - How can we verify that these equations accurately represent the relationship? (substitute the values in for x and y and see if the equation holds true)
  - Since there are 128 fluid ounces in a gallon, students may write the equation $128f = g$ as opposed to $f = 128g$, where f is the number of fluid ounces and g is the number of gallons. How can we use the strategy of checking values in this case? For example, if there are 2 fluid ounces, the equation $128f = g$ would indicate there were 256 gallons, which is not a reasonable answer.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In the lesson, students used a number of ways to reason about the relationship between quantities and to write an equation to represent that relationship. The goal is for students to summarize the reasoning strategies. Facilitate a discussion using the following questions. As students respond, make connections to examples from the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "What reasoning strategies did you use while working with the problems in today's lesson?" (made tables, looked for patterns, tried different numbers and operations)
- "How does a table help in describing relationships?" (The table helps to see how the quantities change. It can also show which quantities are related.)
- "How does trying different numbers help in describing relationships?" (It helps to see how changing one quantity affects the other. It can also help in understanding the operations used to connect the two quantities.)

PLANNING NOTES

Student Lesson Summary and Glossary

Sometimes, the relationship between two quantities is easy to see. For instance, we know that the perimeter of a square is always 4 times the side length of the square. If P represents the perimeter and s the side length, then the relationship between the two measurements (in the same unit) can be expressed as $P = 4s$, or $s = \frac{P}{4}$.

Other times, the relationship between quantities might take a bit of work to figure out—by doing calculations several times or by looking for a pattern. Here are two examples.

- A plane departed from New Orleans and is heading to San Diego. The table shows its distance from New Orleans, x, and its distance from San Diego, y, at some points along the way.

Miles from New Orleans	Miles from San Diego
100	1,500
300	1,300
500	1,100
	1,020
900	700
1,450	
x	y

What is the relationship between the two distances? Do you see any patterns in how each quantity is changing? Can you find out what the missing values are?

Notice that every time the distance from New Orleans increases by some number of miles, the distance from San Diego decreases by the same number of miles, and that the sum of the two values is always 1,600 miles.

The relationship can be expressed with any of these equations:

$x + y = 1,600$

$y = 1,600 - x$

$x = 1,600 - y$

- In the following pattern, the number of squares is related to the figure number.

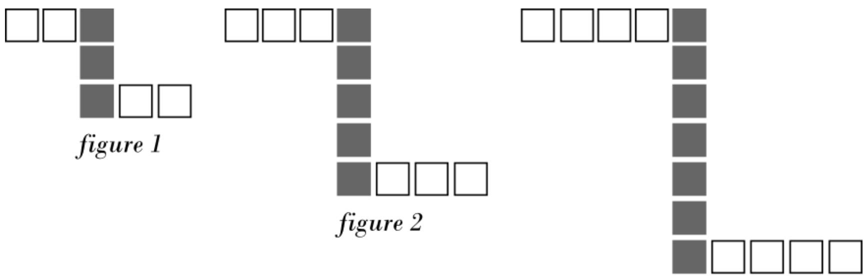


figure 1 *figure 2* *figure 3*

There are a variety of ways to look for a pattern and express the relationship. One way is with a table.

Figure number	Number of squares
1	7
2	11
3	15
4	
5	

Do you notice a pattern?

- As the figure number increases by 1, the number of squares increases by 4.
- $4(1) + 3 = 7$ and $4(2) + 3 = 11$ and $4(3) + 3 = 15$

We can generalize that the number of squares is 4 times the figure number plus 3. This can be expressed by the equation $s = 4f + 3$ where f is the figure number and s is the number of squares.

Cool-down: Salary and Deposit (*5 minutes*)

Addressing: NC.M1.A-CED.2

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

The table below represents the relationship between two quantities.

Monthly salary in dollars, x	872	998	1,015	2,110
Amount deposited in dollars, y	472	598	615	1,710

1. Describe in words how the two quantities in the table are related.
2. Write an equation to represent the relationship between the two quantities.

Student Reflection:

I felt really confident today when I was____.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students had opportunities to share their diagrams and thinking during whole-class discussion? How did you select these students?

Practice Problems

1. A landscaping company is delivering crushed stone to a construction site. The table shows the total weight in pounds, W, of n loads of crushed stone.

   Which equation could represent the total weight, in pounds, for n loads of crushed stone?

Number of loads of crushed stone	Total weight in pounds
0	0
1	$2,000$
2	$4,000$
3	$6,000$

   a. $W = \frac{6,000}{n}$
   b. $W = 6,000 - 2,000n$
   c. $W = 2,000n$
   d. $W = n + 2,000$

2. Tyler needs to complete this table for his consumer science class. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Number of teaspoons	Number of tablespoons	Number of cups
		2
36	12	
	48	3

   a. Complete the missing values in the table.

   b. Write an equation that represents the number of teaspoons, t, contained in a cup, C.

3. The volume of dry goods, like apples or peaches, can be measured usings bushels, pecks, and quarts. A bushel contains 4 pecks, and a peck contains 8 quarts.

   What is the relationship between number of bushels, b, and the number of quarts, q? If you get stuck, try creating a table.

4. Elena has $225 in her bank account. She takes out $20 each week for w weeks. After w weeks she has d dollars left in her bank account.

   Write an equation that represents the amount of money left in her bank account after w weeks.

   (From Unit 2, Lesson 2)

5. Priya is hosting a poetry club meeting this week and plans to have fruit punch and cheese for the meeting. She is preparing 8 ounces of fruit punch per person and 2 ounces of cheese per person. Including herself, there are 12 people in the club.

   A package of cheese contains 16 ounces and costs $3.99. A one-gallon jug of fruit punch contains 128 ounces and costs $2.50. Let p represent number of people in the club, f represent the ounces of fruit punch, c represent the ounces of cheese, and b represent Priya's budget in dollars.

   (*continued*)

Select **all** of the equations that could represent the quantities and constraints in this situation.

a. $f = 8 \cdot 12$
b. $c = 2 \cdot 3.99$
c. $2 \cdot 3.99 + 2.50 = b$
d. $2p = c$
e. $8f + 2c = b$

(From Unit 2, Lesson 2)

6. The data show the number of free throws attempted by a team in its first 10 games.

2 11 11 11 12 12 13 14 14 15

The median is 12 attempts, and the mean is 11.5 attempts. After reviewing the data, it is determined that 2 should not be included, since that was an exhibition game rather than a regular game during the season.

a. What happens to the median if the 2 (attempts in first game) is removed from the data set?

b. What happens to the mean if the 2 (attempts in first game) is removed from the data set?

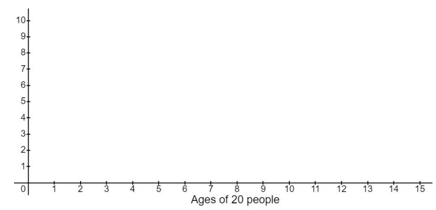
(From Unit 1)

7. The standard deviation for a data set is 0. What can you conclude about the data?

(From Unit 1)

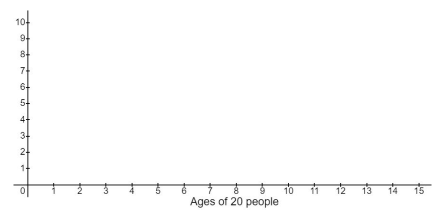
8. Draw a histogram of a data distribution representing the ages of 20 people for which the median and the mean would be approximately the same.[1]

(From Unit 1)



9. Draw a histogram of a data distribution representing the ages of 20 people for which the median is noticeably less than the mean.[2]

(From Unit 1)



10. Complete the table so that each pair of numbers makes the equation true.

a. $m = 2n + 1$

n	m
3	
	5
	12

b. $d = \frac{16}{e}$

e	d
4	
-3	
	2

(Addressing NC.6.EE.9)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

Lesson 4: Equations and Their Solutions

PREPARATION

Lesson Goals	Learning Targets
• Explain (orally and in writing) the meaning of solutions to equations in one variable and two variables. • Find solutions to equations in one variable and in two variables by reasoning about the relationships in context. • Interpret solutions to equations in one variable and in two variables.	• I can explain what it means for a value or pair of values to be a solution to an equation. • I can find solutions to equations by reasoning about a situation or by using algebra.

Lesson Narrative

In middle school, students learned that a solution to an equation is a value or values that make the equation true. In this lesson, they revisit what they learned about solutions to equations in one variable and two variables. They also continue to practice modeling relationships with equations and to make sense of equations and their solutions in context (MP2, MP4).

Students verify and find solutions to given equations by checking if the values satisfy the equations and by reasoning. Some students may choose to solve equations algebraically or by performing certain sequences of steps they learned in middle school, but students are not expected to rely on algebraic methods to answer questions here. A little later in the unit, students will take a close look at the moves for rewriting and solving equations.

In the next unit, students will revisit the idea that coordinate pairs that are on a graph of an equation in two variables are solutions to the equation.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.7.EE.1: Apply properties of operations as strategies to: • Add, subtract, and expand linear expressions with rational coefficients. • Factor linear expression with an integer GCF.	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L4 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.1

The purpose of this bridge is for students to encounter typical errors with signed numbers, operations, and properties. They are tasked with identifying which strategies are correct and, for those that are not, describing the error that was made. This will support students later in this unit (in Lesson 5) as they identify equivalent expressions. This task is also useful for students who may have struggled with questions 3 or 6 from Check Your Readiness.

Student Task Statement

Some students are trying to write an expression with fewer terms that is equivalent to $8-3(4-9x)$. For each student, circle if you agree or disagree and share your reasoning, including finding and describing any errors that may have occurred.[1]

Noah says, "I worked the problem from left to right and ended up with $20-45x$." $8-3(4-9x)$ $5(4-9x)$ $20-45x$	Lin says, "I started inside the parentheses and ended up with $23x$." $8-3(4-9x)$ $8-3(-5x)$ $8+15x$ $23x$	Jada says, "I used the distributive property and ended up with $-4+27x$." $8-3(4-9x)$ $8-(12-27x)$ $8-12-(-27x)$ $-4+27x$	Andre says, "I also used the distributive property, but I ended up with $-4-27x$." $8-3(4-9x)$ $8-12-27x$ $-4-27x$
Agree Disagree	Agree Disagree	Agree Disagree	Agree Disagree

DO THE MATH

PLANNING NOTES

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: What Is a Solution? (*10 minutes*)

Instructional Routine: Collect and Display (MLR2)
Addressing: NC.M1.A-REI.3

This warm-up prompts students to recall what they know about the solution to an equation in one variable. Students interpret a given equation in the context of a situation, explain why certain values are not solutions to the equation, and then find the value that is the solution.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 1 minute of quiet think time and then 2–3 minutes to share their thinking with their partner.

Student Task Statement

A granola bite contains 27 calories. Most of the calories come from c grams of carbohydrates. The rest come from other ingredients. One gram of carbohydrate contains 4 calories.

The equation $4c + 5 = 27$ represents the relationship between these quantities.

1. What could the 5 represent in this situation?
2. Priya said that neither 8 nor 3 could be the solution to the equation. Explain why she is correct.
3. Find the solution to the equation.

Step 2

Facilitate a whole-class discussion.

- Ask students how they know that 8 and 3 are not solutions to the equation and how they found the solution. Highlight strategies that are based on reasoning about what values make the equation true.
- Ask students: "In general, what does a solution to an equation mean?" Make sure students recall that the solution to an equation in one variable is a value for the variable that makes the equation a true statement.
  - *Collect and Display* any phrases that may be useful for students to use as part of their first or second drafts during the *Stronger and Clearer Each Time* routine in the following activity.

DO THE MATH	PLANNING NOTES

Activity 1: Calories from Protein and Fat (*20 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)
Building Towards: NC.M1.A-CED.2; NC.M1.A-REI.10

In the previous activity, students recalled what it means for a number to be a solution to an equation in one variable. In this activity, they review the meaning of a solution to an equation in two variables.

Step 1

- Keep students in pairs.
- Display the task prompt for all to see.
- Ask students to try and identify a pair of possible values for grams of protein and grams of fat.
  - Have students record each pair they try and whether the pair is possible or not.

Advancing Student Thinking: If students struggle to get started, provide a pair of values such as 5 grams of protein and 2 grams of fat. Ask them to see if the pair is possible.

Student Task Statement

One gram of protein contains 4 calories. One gram of fat contains 9 calories. A snack has 60 calories from p grams of protein and f grams of fat.

The equation $4p + 9f = 60$ represents the relationship between these quantities.

Identify pairs of values that could be the number of grams of protein and number of grams of fat in the snack. Record the pairs you try in a table like the one shown below. Be prepared to explain your reasoning.

Number of grams of protein	Number of grams of fat	Is this a possible pair of values for the 60 calorie snack? Yes or No

Step 2

Facilitate a whole-class discussion.

- Ask students to share at least one pair that they tried and whether or not it was possible.
- Record responses into two groups: possible solutions and not-possible solutions.
  - If no one suggests a possible solution, ask students to try 6 grams of protein and 4 grams of fat.
- If time allows, ask students, “If there were 10.5 grams of protein, how could you find the number of grams of fat?” (Substitute 10.5 for p and solve the equation $4(10.5) + 9f = 60$. The solution is 2 grams of fat.)

Step 3

- Use the *Stronger and Clearer Each Time* routine to set students up to clarify their reasoning and their language about the meaning of a solution. Ask students to jot down a first draft response to the question, “In this situation, what does a solution to the equation $4p + 9f = 60$ tell us?” Refer back to any useful phrases captured during the warm-up discussion.

- After 1 minute of individual writing time, arrange students into pairs with a new partner to give and receive feedback on their first draft thinking. Feedback should take the form of clarifying questions and additional ideas. Have students take turns sharing their ideas and receiving feedback, then repeat the process with another partner. Allow 1–2 minutes for each round and encourage students to take notes for themselves on useful ideas from each conversation.

- Finally, have students improve their first draft responses. Remind them to use ideas from their partner conversations to make their own response stronger and clearer.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

To summarize the lesson, refer back to the activity about protein and fat. Remind students that a gram of protein has 4 calories and a gram of fat has 9 calories.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. • "What does the equation $4p + 9f = 110$ tell us about the calories in a snack?" (It has 110 calories from some grams of protein and some grams of fat.) • "In this situation, what does it mean to find a solution to the equation?" (To find a combination of grams of protein and fat that produce 110 calories.) • "Is the combination of 11 grams of protein and 5 grams of fat a solution to the equation? Why or why not?" (No, they don't add up to 110 calories. Substituting 11 for p and 5 for f into the equation doesn't lead to a true equation.) • "Consider the equation $4(5) + 9f = 110$. What does it tell us about the snack?" (The snack has 5 grams of protein and a total of 110 calories.) • "What does it mean to solve this equation?" (To find the number of grams of fat that, when combined with 5 grams of calories, give a total of 110 calories. To find the value of f that would make the equation true.)	PLANNING NOTES

Student Lesson Summary and Glossary

An equation that contains only one unknown quantity or one quantity that can vary is called an “equation in one variable.”

For example, the equation $8h+50=250$ represents the fact that Clare began with $\$50$ saved, worked for h hours at a job that paid $\$8$ per hour, and now has $\$250$.

This is an equation in one variable, because h is the only quantity that we don't know. To solve this equation means to find a value of h that makes the equation true.

In this case, 25 is the solution because substituting 25 for h in the equation results in a true statement.

$$\begin{aligned} 8h+50&=250\\ 8(25)+50&=250\\ 200+50&=250\\ 250&=250 \end{aligned}$$

An equation that contains two unknown quantities or two quantities that vary is called “an equation in two variables.” A solution to such an equation is *a* pair of numbers that makes the equation true.

Suppose Tyler spends $\$45$ on T-shirts and socks. A T-shirt costs $\$10$, and a pair of socks costs $\$2.50$. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, we can represent this situation with the equation:

$$10t+2.50p=45$$

This is an equation in two variables. More than one pair of values for t and p make the equation true.

$t=3$ and $p=6$

$$\begin{aligned} 10(3)+2.50(6)&=45\\ 30+15&=45\\ 45&=45 \end{aligned}$$

$t=4$ and $p=2$

$$\begin{aligned} 10(4)+2.50(2)&=45\\ 40+5&=45\\ 45&=45 \end{aligned}$$

$t=2$ and $p=10$

$$\begin{aligned} 10(2)+2.50(10)&=45\\ 20+25&=45\\ 45&=45 \end{aligned}$$

In this situation, one constraint is that the combined cost of shirts and socks must equal $\$45$. Solutions to the equation are pairs of t and p values that satisfy this constraint.

Combinations such as $t=1$ and $p=10$ or $t=2$ and $p=7$ are *not* solutions because they don’t meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.

Cool-down: Box of T-shirts (*5 minutes*)

Addressing: NC.M1.A-REI.3	**Building Towards:** NC.M1.A-CED.2
Cool-down Guidance: More Chances Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.	

Cool-down

An empty shipping box weighs 250 grams. The box is then filled with T-shirts. Each T-shirt weighs 132.5 grams.

The equation $W = 250 + 132.5T$ represents the relationship between the quantities in this situation, where W is the weight, in grams, of the filled box and T is the number of shirts in the box.

1. Name two possible solutions to the equation $W = 250 + 132.5T$. What do the solutions mean in this situation?
2. Consider the equation $2,900 = 250 + 132.5T$. Solve the equation and explain what the solution means in this situation.

Student Reflection:

Why is what we learned today important?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In an upcoming lesson, students will be justifying the steps in solving equations. What do you notice in their work from today's lesson that you might leverage in that future lesson?

Practice Problems

1. An artist is selling children's crafts. Necklaces cost \$2.25 each, and bracelets cost \$1.50 each. Select **all** the combinations of necklaces and bracelets that the artist could sell for exactly \$12.00.

    a. 5 necklaces and 1 bracelet
    b. 2 necklaces and 5 bracelets
    c. 3 necklaces and 3 bracelets
    d. 4 necklaces and 2 bracelets
    e. 3 necklaces and 5 bracelets
    f. 6 necklaces and no bracelets
    g. No necklaces and 8 bracelets

2. Diego is collecting dimes and nickels in a jar. He has collected \$22.25 so far. The relationship between the numbers of dimes and nickels, and the amount of money in dollars, is represented by the equation $0.10d + 0.05n = 22.25$.

    Select **all** the values (d, n) that could be solutions to the equation.

    a. $(0, 445)$
    b. $(0.50, 435)$
    c. $(233, 21)$
    d. $(118, 209)$
    e. $(172, 101)$

3. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat 4 students, or vans, which can seat 6 students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c, and number of vans, v, that can transport exactly 80 students.

    Select **all** statements that are true about the situation.

    a. If 12 cars go, then 2 vans are needed.
    b. $c = 14$ and $v = 4$ are a pair of solutions to the equation.
    c. If 6 cars go and 11 vans go, there will be extra space.
    d. 10 cars and 8 vans aren't enough to transport all the students.
    e. If 20 cars go, no vans are needed.
    f. 8 vans and 8 cars are numbers that meet the constraints in this situation.

4. The drama club is printing T-shirts for its members. The printing company charges a certain amount for each shirt plus a setup fee of \$40. There are 21 students in the drama club.

    a. If there are 21 students in the club and the t-shirt order costs a total of \$187, how much does each t-shirt cost? Show your reasoning.

    b. The equation $201.50 = f + 6.50(21)$ represents the cost of printing the shirts at a second printing company. Find the solution to the equation and state what it represents in this situation.

5. Identify the error in generating an expression equivalent to $4 + 2x - \frac{1}{2}(10 - 4x)$. Then correct the error.[2]

$$\begin{gathered} 4 + 2x + -\tfrac{1}{2}(10 + -4x) \\ 4 + 2x + -5 + 2x \\ 4 + 2x - 5 + 2x \\ -1 \end{gathered}$$

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

6. Kiran's family is having people over to watch a football game. They plan to serve sparkling water and pretzels. They are preparing 12 ounces of sparkling water and 3 ounces of pretzels per person. Including Kiran's family, there will be 10 people at the gathering. A bottle of sparkling water contains 22 ounces and costs $1.50. A package of pretzels contains 16 ounces and costs $2.99. Let n represent number of people watching the football game, s represent the ounces of sparkling water, p represent the ounces of pretzels, and b represent Kiran's budget in dollars. Which equation best represents Kiran's budget?

   a. $12s + 3p = b$
   b. $12 \cdot 10 + 3 \cdot 10 = b$
   c. $1.50s + 2.99p = b$
   d. $1.50 \cdot 6 + 2.99 \cdot 2 = b$

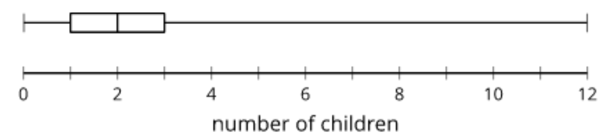
   (From Unit 2, Lesson 2)

7. The speed of an object can be found by taking the distance it travels and dividing it by the time it takes to travel that distance. An object travels 100 feet in 2.5 seconds. Let the speed, S, be measured in feet per second.

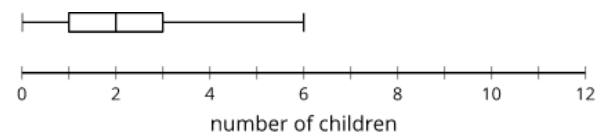
   Write an equation to represent the relationship between the three quantities (speed, distance, and time).

   (From Unit 2, Lesson 2)

8. The box plot represents the distribution of the number of children in 30 different families.



   After further examination, the value of 12 is removed for having been recorded in error. The box plot represents the distribution of the same data set, but with the maximum, 12, removed.



   The median is 2 children for both plots.

   a. Explain why the median remains the same when 12 was removed from the data set.

   b. When 12 is removed from the data set, does the mean remain the same? Explain your reasoning.

   (From Unit 1)

9. The number of points Jada's basketball team scored in their games has a mean of about 44 and a standard deviation of about 15.7 points.

   Interpret the mean and standard deviation in the context of Jada's basketball team.

   (From Unit 1)

10. Noah says that $9x - 2x + 4x$ is equivalent to $3x$ because the subtraction sign tells us to subtract everything that comes after $9x$. Elena says that $9x - 2x + 4x$ is equivalent to $11x$ because the subtraction only applies to $2x$.

   Do you agree with either of them? Explain your reasoning.[3]

   (Addressing NC.7.EE.1; NC.7.EE.3)

[3] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 5: Equivalent Equations

PREPARATION

Lesson Goals	Learning Targets
• Comprehend that "equivalent equations" are equations that have exactly the same solutions, and that multiple equivalent equations can represent the same relationship. • Determine and explain (orally and in writing) whether two equations are equivalent. • Identify operations that can be performed on an equation to create equivalent equations.	• I can tell whether two expressions are equivalent and explain why or why not. • I know and can identify the moves that can be made to transform an equation into an equivalent one. • I understand what it means for two equations to be equivalent, and how equivalent equations can be used to describe the same situation in different ways.

Lesson Narrative

In middle school, students learned that two expressions are equivalent if they have the same value for all values of the variables in the expressions. They wrote equivalent expressions by applying properties of operations, combining like terms, or rewriting parts of an expression.

In this lesson, students learn that **equivalent equations** are equations with the exact same solutions. Students see that the moves that generate equivalent expressions (for example, applying the distributive property or combining like terms) can also create equivalent equations. Additionally, an equivalent equation can be created by adding the same number to both sides or multiplying both sides by the same non-zero number. Students have seen moves like this before, when solving one-variable equations in middle school. What is new here is an awareness that each of the equations created as a part of the solving process is *equivalent* to the original equation.

Students also regard equivalent equations as synonymous statements about a relationship. They use context to interpret the solution to equivalent equations and to think about why it makes sense that equivalent equations have the same solution. In doing so, students reason abstractly and quantitatively (MP2).

The emphasis of this lesson is on equations in one variable.

What do you hope to learn about your students in this lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.4: Identify when two expressions are equivalent and justify with mathematical reasoning	**NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-SSE.1:** Interpret expressions that represent a quantity in terms of its context.

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
- **Activity 1** (*25 minutes*)
  - Equation Cards card sort (print 1 copy per every 2 students and cut up in advance)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L5 Cool-down (print 1 copy per student)

LESSON

Warm-up: Two Expressions (*10 minutes*)

Instructional Routine: Notice and Wonder	
Building On: NC.6.EE.4	**Building Towards:** NC.M1.A-REI.1

The purpose of this warm-up is to help students recall what it means for two expressions to be equivalent. The given expressions are in forms that are unfamiliar to students but are not difficult to evaluate for integer values of the variable. This is by design—to pique students' curiosity while keeping the mathematics accessible.

Step 1

- Display the two expressions. Use the *Notice and Wonder* routine and ask students, “What do you notice? What do you wonder?”
- Give students a minute to think of things they notice and things they wonder and then share them with a partner.
- Ask several students to share things they noticed and things they wondered; record students’ noticings and wonderings for all to see.

Step 2

- Assign the first expression to one half of the class and the second expression to the other half.
- Give students a couple of minutes to evaluate their assigned expression for the values of n listed.

***Advancing Student Thinking*:** When evaluating their expression, some students may perform the operations in an incorrect order. For example, when finding the value of $8-3 \cdot 2$ in the second expression, they may find $8-3$ and then multiply by 2. Ask them whether the subtraction or multiplication should be performed first. Remind them about the order of operations, as needed.

Student Task Statement

Your teacher will assign you one of these expressions:

$$\frac{n^2-9}{2(4-3)} \quad \textit{or} \quad (n+3)\cdot\frac{(n-3)}{8-3\cdot 2}$$

Evaluate your expression when n is:

1. 5
2. 7
3. 13
4. -1

Step 3

- Ask a few students from each group for their results.
- Ask students what they wonder about the results.
  - Students are likely curious if the values of the two expressions will be the same for other values of n.
  - If they noticed that all the given values of n are odd numbers, they might wonder if even values of n would give the same result.
  - If time permits, consider allowing students to try evaluating the expressions using a value of their choice.

Step 4

- Facilitate a whole-class discussion to reintroduce equivalent expressions. Discuss questions such as:
  - "Were you surprised that, for each value of n, these expressions have the same value?"
  - "If (or when) you tried using other values of n, what did you find?"
  - "Do you think that the two expressions will have the same value no matter what value of n is used? How do you know?"
- Tell students that it would be impossible to check every value of n to see if the expressions would give the same value. There are, however, ways to show that these expressions must have the same value for any value of n. Expressions that are equal no matter what value is used for the variable are called "equivalent expressions."
- Remind students that in middle school they had seen simpler equivalent expressions. For example, they know that $3(x+5)$ is equivalent to $3x+15$ by the distributive property (without trying different values of x).
- Explain to students that they'll learn more about how to identify or write equivalent expressions and about equivalent equations in this unit.

RESPONSIVE STRATEGIES

Create a display of important terms and vocabulary. Keep this display visible throughout the remainder of the unit. Invite students to suggest language or diagrams to include that will support their understanding of combining like terms, and the commutative, associative, and distributive properties.

Supports accessibility for: Conceptual processing; Language

DO THE MATH	PLANNING NOTES

Activity 1: What's Acceptable? (*25 minutes*)

Instructional Routines: Card Sort; Take Turns; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.A-REI.1; NC.M1.A-SSE.1

This activity develops the idea of equivalent equations and does so in the context of a situation. Students pay attention to the moves that create equations with the same solution and those that don't. They also make sense of both the moves and the solutions in terms of the given situation. The work here gives students opportunities to reason concretely and abstractly (MP2). In an upcoming lesson, they will think about why these moves lead to equations with the same solution.

Step 1

- Display the following equations: $2x+4=10$, $6x+12=30$, and $2x=6$.
- Solicit from students some thoughts on the following questions. (It is not necessary to resolve the questions at the moment.)
  - "Consider the three equations. Are they all equivalent? Why or why not?"
  - "What do you think it means for equations to be equivalent?"
- After students have had a chance to consider these questions, move to Step 2.

Step 2

- Give students the choice to work individually or in pairs to solve question 1.

Student Task Statement

1. Are any of these equations equivalent to one another? If so, which ones? Explain your reasoning.

   $3a+6=15$
   $3a=9$
   $a+2=5$
   $\frac{1}{3}a=1$

- After a couple minutes, have students share their responses.
- Next, emphasize two main points:
  - Equivalent equations have exactly the same solutions or exactly the same values that make each of the equations true. All of the equations in question 1 are equivalent because they have 3 as the solution and they all have no other solutions.
  - Suppose we start with a true equation, where the two sides are equal. If we perform the same operation to both sides of the equation and get a new equation where the two sides are also equal, we can say that the two equations are equivalent. For instance:
    - Subtracting both sides of $3a+6=15$ by 6 gives $3a=9$. If $3a+6$ and 15 are equal, then the expressions or numbers we get by subtracting 6 from each one are also equal. We can conclude that $3a+6=15$ and $3a=9$ are equivalent.
    - Dividing both sides of $3a+6=15$ by 3 gives $a+2=5$. If $3a+6$ is equal to 15, the result of dividing $3a+6$ by 3 is equal to dividing 15 by 3. We can conclude that $3a+6=15$ and $a+2=5$ are equivalent.
- Ask students:
  - "How can we show that $\frac{1}{3}a=1$ is equivalent to $3a=9$?" (Multiplying both sides of $\frac{1}{3}a=1$ by 9 gives $3a=9$.)
  - "How can we show that $a+2=5$ is equivalent to $\frac{1}{3}a=1$?" (Subtracting 2 from both sides of $a+2=5$ and then multiplying both sides of the resulting equation, $a=3$, by $\frac{1}{3}$ gives $\frac{1}{3}a=1$.)

If no students notice that we have made these moves when solving equations, bring it to their attention. Highlight that solving an equation essentially involves writing a series of equivalent equations that eventually isolates the variable on one side.

Step 3

- Provide students access to calculators.
- Display the opening paragraph about Noah's jeans along with the equation. Ask students to explain how the equation represents Noah's purchase.
- Give students a couple of minutes to discuss the first question with their partner and ask them to pause for a whole-class discussion. Once students understand that the solution to the equation is the original price of a pair of jeans and that substituting 60 for x makes the equation true, move on to the rest of the activity.
- Pass out the Equation Cards to each group. Tell students that they will use a *Card Sort* routine to match cards with equations that are related to the original equation to cards that interpret the equation in terms of the situation.

> ***Why This Routine?*** A *Card Sort* provides opportunities to attend to mathematical connections using representations that are already created, instead of expending time and effort generating representations. It gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

- After matching the equations and interpretations, students are to determine what move was made to the original equation and then check if the equation has the same solution as the original.
  - Instruct students to use the *Take Turns* routine, with one partner suggesting an interpretation card and the other, if in agreement, suggesting what was done to the original equation to get the new equation. Then work together to confirm whether or not 60 is the solution to the new equation. This will encourage equitable engagement in the mathematical thinking.

Student Task Statement

Noah is buying a pair of jeans and using a coupon for 10% off. The total price is $56.70, which includes $2.70 in sales tax.

Noah's purchase can be modeled by the equation:

$x - 0.1x + 2.70 = 56.70$

2. Discuss with a partner:
    a. What does the solution to the equation mean in this situation?
    b. How can you verify that 70 is not a solution, but 60 is the solution?
3. Your teacher will give you a set of six cards. Three of the cards have equations that are related to $x - 0.1x + 2.70 = 56.70$. The other three cards have interpretations in terms of Noah's purchase. Match the cards together.
4. The table below shows the three equations from the card sort. Each equation is a result of performing one or more moves on the original equation describing Noah's purchase. For each equation:
    a. What move was made?
    b. Check if 60 is the solution to the equation.

Original Equation: $x - 0.1x + 2.70 = 56.70$			
Equation	**Card #**	**What was done to the original equation to get this equation?**	**Is 60 the solution to the equation? (yes or no)**
$100x - 10x + 270 = 5{,}670$			
$x - 0.1x = 54$			
$0.9x + 2.70 = 56.70$			

5. Andre said he found two more equations that are equivalent to the original. Mai says they aren't equivalent because 60 isn't the solution to the equations. Is Mai correct? If so, what moves did Andre make that weren't "acceptable"?

   $2(x - 0.1x + 2.70) = 56.70$ $x - 0.1x = 59.40$

Step 4

- Display the original equation and the three equations for all to see.
- Invite students to share what was done to the original equation to get each of those equations and whether they have the same solution. Along the way, compile a list of moves that lead to equations with the same solution, such as:
    - adding or multiplying both sides of the equal sign by the same number
    - applying properties of operations (commutative, associative, or distributive)
    - combining like terms

(The lists don't need to be comprehensive, as students will examine these moves more closely later.)

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Discussion Supports (MLR8)

- Have students share their responses to #5 and add a list of moves that lead to different solutions, such as:
  - adding or multiplying a different number to the two sides, or performing an operation to only one side
  - adding different expressions to the two sides
  - performing different operations on each side
- Ask students to observe the list and see what kinds of moves produced the first three equations compared to the moves that Andre made.

Are You Ready For More?

Here is a puzzle:

$$\begin{aligned} m+m &= N \\ N+N &= p \\ m+p &= Q \\ p+Q &= \,? \end{aligned}$$

Which expressions could be equal to $p+Q$?

a. $2p+m$

b. $4m+N$

c. $3N$

d. $9m$

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students learn that equivalent equations are equations with the exact same solutions. To help students connect the various ideas in this lesson and articulate their understanding, facilitate a discussion using these questions.

Choose whether students should first have an opportunity to reflect on the following questions in their workbooks or talk through them with a partner. Determine what questions will be prioritized in the full class discussion.

- "How would you explain 'equivalent equations' to a classmate who is absent today?"
- "The equation $5y = 6$ represents purchasing 5 tubs of yogurt for \$6. In this equation, what does the solution represent?" (the cost of one tub of yogurt)
- "Which of these equations are equivalent to the equation $5y = 6$ (about the yogurt)? How do you know?"

  a. $15y = 18$

  b. $5y = 12$

  c. $5y + 4 = 10$

  d. $5y - 1 = 3$

  (Equations "a" and "c" are equivalent to the original equation because they have the same solution. There is an acceptable move that was done to the original equation to get to these equivalent equations: multiplying each side of the equal sign by 3, and adding 4 to each side of the equation.)
- "For the equations that you think are equivalent, what do they mean in the context of the yogurt purchase?" (The equation $15y = 18$ can be interpreted as: buying 3 times as many tubs of yogurt costs 3 times as much. Equation "c" can be interpreted as: the total cost of 5 tubs of yogurt and something else that costs \$4 is \$10.)

PLANNING NOTES

Student Lesson Summary and Glossary

Suppose we bought two packs of markers and a $\$0.50$ glue stick for $\$6.10$. If m is the dollar cost of one pack of markers, the equation $2m + 0.50 = 6.10$ represents this purchase. The solution to this equation is 2.80.

Now suppose a friend bought six of the same packs of markers and three $\$0.50$ glue sticks, and paid $\$18.30$. The equation $6m + 1.50 = 18.30$ represents this purchase. The solution to this equation is also 2.80.

We can say that $2m + 0.50 = 6.10$ and $6m + 1.50 = 18.30$ are **equivalent equations** because they have exactly the same solution. Besides 2.80, no other values of m make either equation true. Only the price of $\$2.80$ per pack of markers satisfies the constraint in each purchase.

Equivalent equations: Equations that have exactly the same solutions.

$$2m + 0.50 = 6.10$$

$$6m + 1.50 = 18.30$$

How do we write equivalent equations like these?

There are certain moves we can perform!

In this example, the second equation, $6m + 1.50 = 18.30$, is a result of multiplying each side of the first equation by 3. Buying 3 times as many markers and glue sticks means paying 3 times as much money. The unit price of the markers hasn't changed.

Here are some other equations that are equivalent to $2m + 0.50 = 6.10$, along with the moves that led to these equations.

- $2m + 0.50 = 6.10$
- $2m + 4 = 9.60$ — Add 3.50 to each side of the original equation.

- $2m + 0.50 = 6.10$
- $2m = 5.60$ — Subtract 0.50 from each side of the original equation.

- $2m + 0.50 = 6.10$
- $\frac{1}{2}(2m + 0.50) = 3.05$ — Multiply each side of the original equation by $\frac{1}{2}$.

- $2m + 0.50 = 6.10$
- $2(m + 0.25) = 6.10$ — Apply the distributive property to rewrite the left side.

In each case:

- The move is acceptable because it doesn't change the equality of the two sides of the equation.
- If $2m + 0.50$ has the same value as 6.10, then multiplying $2m + 0.50$ by $\frac{1}{2}$ and multiplying 6.10 by $\frac{1}{2}$ keep the two sides equal.
- Only $m = 2.80$ makes the equation true. Any value of m that makes an equation false also makes the other equivalent equations false. (Try it!)

These moves—applying the distributive property, adding the same amount to both sides, dividing each side by the same number, and so on—might be familiar because we have performed them when solving equations. Solving an equation essentially involves writing a series of equivalent equations that eventually isolates the variable on one side.

Not all moves that we make on an equation would create equivalent equations, however!

For example, if we subtract 0.50 from the left side but add 0.50 to the right side, the result is $2m = 6.60$. The solution to this equation is 3.30, not 2.80. This means that $2m = 6.60$ is *not* equivalent to $2m + 0.50 = 6.10$.

Cool-down: Box of Beans and Rice (*5 minutes*)

Addressing: NC.M1.A-REI.1

Cool-down Guidance: More Chances
If the majority of students struggle with this cool-down, consider building 3 minutes into Step 2 of Activity 2 in Lesson 6. If students struggled to find all of the equivalent equations, then show the equations as a sequence—the original equation, followed by equation b, then equation e—and ask, “How do we know these are equivalent?” Or consider showing the original equation with equation c directly underneath. If multiple students identified incorrect equations as equivalent, consider showing the original equation with equations a or d directly underneath and asking, "What happened?" or "Did the same thing happen to both sides?” If only a few students struggle with this cool-down, then those are the students to support during Activity 2 in Lesson 6.

Students may continue to have access to calculators.

Cool-down

A cardboard box, which weighs 0.6 pounds when empty, is filled with 15 bags of beans and a 4-pound bag of rice. The total weight of the box and the contents inside it is 25.6 pounds. One way to represent this situation is with the equation $0.6 + 15b + 4 = 25.6$.

1. In this situation, what does the solution to the equation represent?
2. Select **all** equations that are also equivalent to $0.6 + 15b + 4 = 25.6$.
   a. $15b + 4 = 25.6$
   b. $15b + 4 = 25$
   c. $3(0.6 + 15b + 4) = 76.8$
   d. $15b = 25.6$
   e. $15b = 21$

Student Reflection:

Doing math work at home or after school makes me feel ___________ because ________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on how comfortable your students are asking questions of you and of each other. What can you do to encourage students to ask questions?

Practice Problems

1. Which equation is equivalent to the equation $6x + 9 = 12$?

   a. $x + 9 = 6$
   b. $2x + 3 = 4$
   c. $3x + 9 = 6$
   d. $6x + 12 = 9$

2. Select **all** the equations that have the same solution as the equation $3x – 12 = 24$.

   a. $15x – 60 = 120$
   b. $3x = 12$
   c. $3x = 36$
   d. $x – 4 = 8$
   e. $12x – 12 = 24$

3. Jada has a coin jar containing n nickels and d dimes worth a total of $3.65. The equation $0.05n + 0.1d = 3.65$ is one way to represent this situation.

   Which equation is equivalent to the equation $0.05n + 0.1d = 3.65$?

   a. $5n + d = 365$
   b. $0.5n + d = 365$
   c. $5n + 10d = 365$
   d. $0.05d + 0.1n = 365$

4. Select **all** the equations that have the same solution as $2x – 5 = 15$.

   a. $2x = 10$
   b. $2x = 20$
   c. $2(x – 5) = 15$
   d. $2x – 20 = 0$
   e. $4x – 10 = 30$
   f. $15 = 5 – 2x$

5. A basketball coach purchases bananas for the players on his team. The table shows total price in dollars, P, of n bananas.

   Which equation could represent the total price in dollars for n bananas?

   a. $P = 0.59n$
   b. $P = 5.90 - 0.59n$
   c. $P = \frac{5.90}{n}$
   d. $P = n + 0.59$

Number of bananas	Total price in dollars
7	4.13
8	4.72
9	5.31
10	5.90

   (From Unit 2, Lesson 3)

6. Kiran is collecting dimes and quarters in a jar. He has collected $10.00 so far and has d dimes and q quarters. The relationship between the numbers of dimes and quarters, and the amount of money in dollars is represented by the equation $0.1d + 0.25q = 10$.

   Select **all** the values (d, q) that could be solutions to the equation.

   a. $(100, 0)$
   b. $(20, 50)$
   c. $(50, 20)$
   d. $(0, 100)$
   e. $(10, 36)$

   (From Unit 2, Lesson 4)

7. Bananas cost \$0.50 each, and apples cost \$1.00 each.

   Select **all** the combinations of bananas and apples that Elena could buy for exactly \$3.50.

   a. 2 bananas and 2 apples
   b. 3 bananas and 2 apples
   c. 1 banana and 2 apples
   d. 1 banana and 3 apples
   e. 5 bananas and 2 apples
   f. 5 bananas and 1 apple

   (From Unit 2, Lesson 4)

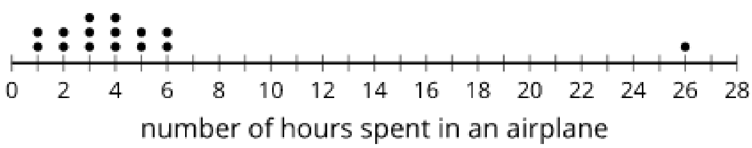
8. The entrepreneurship club is ordering potted plants for all 36 of its sponsors. One store charges \$8.50 for each plant plus a delivery fee of \$20. The equation $320 = x + 7.50(36)$ represents the cost of ordering potted plants at a second store.

   What does the x represent in this situation?

   a. the cost for each potted plant at the second store
   b. the delivery fee at the second store
   c. the total cost of ordering potted plants at the second store
   d. the number of sponsors of the entrepreneurship club

   (From Unit 2, Lesson 4)

9. The number of hours spent in an airplane on a single flight is recorded on a dot plot. The mean is 5 hours; the median is 4 hours, and the IQR is 3 hours. The value 26 hours is an outlier that should not have been included in the data.



   When the outlier is removed from the data set:

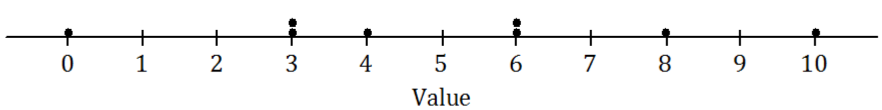
   a. What is the mean?

   b. What is the median?

   c. What is the IQR?

   (From Unit 1)

10. Look at the dot plot below.[1]



   a. Estimate the mean of this data set.

   b. Remember that the standard deviation measures a typical deviation from the mean. The standard deviation of this data set is either 3.2, 6.2, or 9,2. Which of these values is correct for the standard deviation?

   (From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Lesson 6: Explaining Steps for Rewriting Equations

PREPARATION

Lesson Goals	Learning Targets
• Explain (orally and in writing) why performing certain operations on an equation may create equivalent equations but performing other operations may not. • Understand that dividing by a variable is not used in solving equations because it can lead to equations that have fewer solutions than the original equation. • Understand that equations that are not true for any value of the variable(s) do not have solutions.	• I can explain why some algebraic moves create equivalent equations but some do not. • I know how equivalent equations are related to the steps of solving equations. • I know what it means for an equation to have no solutions and can recognize such an equation.

Lesson Narrative

In an earlier lesson, students learned that equivalent equations are equations with the same solution. They practiced identifying and writing equivalent equations by performing some acceptable moves. Students also recognized that, when solving one-variable equations, they are essentially writing a series of equivalent equations that leads to the solution.

This lesson serves two main goals. The first goal is to further develop the idea of equivalent equations. Students think about and articulate how they know that the equations produced using acceptable moves are indeed equivalent. The process is an opportunity to practice constructing logical arguments and critiquing the reasoning of others (MP3).

The second goal is to refamiliarize students with equations with no solutions (which they encountered in grade 8), and with a move that might appear acceptable (dividing each side of an equation by the same variable expression) but would in fact lead to the wrong conclusion.

In what ways will you encourage students to persevere in this lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.5: Use substitution to determine whether a given number in a specified set makes an equation true.	**NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U2.L6 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.5

The purpose of this bridge is to give students an opportunity to practice checking whether a particular value is a solution to an equation, and to recall properties of operations and equality that preserve the solution set of an equation. This will be useful later in the lesson when students consider when and why equations have the same solution. It also gives students an opportunity to encounter problems in which the given value is not a solution to the equation, which will be useful in Activity 2 of this lesson.

Student Task Statement

1. Is $x = 4$ a solution to:
    a. $x(4+3) = 28$
    b. $4x + 3x = 28$
2. Is $x = 5$ a solution to:
    a. $x - 4 = -1$
    b. $4 - x = -1$

DO THE MATH	PLANNING NOTES

Warm-up: Could It Be Zero? (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.A-REI.1

This *Math Talk* encourages students to rely on the structure of equations, properties of operations, and what they know about solutions to equations to mentally solve problems. It also prompts students to recall that dividing a number by 0 leads to an undefined result, preparing them for the work later in the lesson. (In that activity, students will consider why dividing by a variable is not considered an acceptable move when writing equivalent equations or solving equations.)

To determine if 0 is a solution to the equations, students could substitute 0 into the expressions and evaluate them. For some equations, however, the answer can be efficiently found by making use of structure (MP7). In explaining their strategies, students need to be precise in their word choice and use of language (MP6).

Why This Routine? A *Math Talk* builds fluency by encouraging students to think about the numbers, shapes, or algebraic expressions and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. While participating in these activities, there is a natural need for students to be precise in their word choice and use of language (MP6). Additionally, a Math Talk often provides opportunities to notice and make use of structure (MP7).

Step 1

- Display one problem at a time.
- Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the *Math Talk*.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Determine if 0 is a solution to each equation.

$$4(x+2)=10$$

$$12-8x=3(x+4)$$

$$5x=\frac{1}{2}x$$

$$\frac{6}{x}+1=8$$

Step 2

- Facilitate a whole-class discussion. Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate __________'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ___________'s strategy?"
  - "Do you agree or disagree? Why?"

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because...." or "I noticed ____ so I...." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 1: Explaining Acceptable Moves (*15 minutes*)

Instructional Routines: Take Turns; Collect and Display (MLR2); Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.A-REI.1

Previously, students saw that certain moves can be made to an equation to create an equivalent equation. In this activity, they deepen that understanding by explaining why, if a given equation is true for a certain value of the variable, performing one of those moves leads to a second equation that is also true for the same variable value. This requires more than simply stating what the moves are and offers students opportunities to take turns giving explanations, listen to and critique a partner's explanations, and construct logical arguments (MP3).

Step 1

- Tell students that they have performed a number of moves to write equivalent equations and solve equations. They will now practice explaining to a partner why those moves are legitimate. Demonstrate what it means to explain or defend the steps rather than simply describing them.
  - For example, ask students: "Consider these equations: $3x-5=16$ and $3x=21$. The first equation is a true statement for a certain value of x. Can you explain why the second equation must also be true for the same value of x?"
  - Explain that an answer such as "adding 5 to each side of the first equation gives the second equation" is a description of the move rather than an explanation for why the second equation must be true.
  - An explanation of why the second equation must be true may sound something like: "We know that $3x-5$ and 16 are equal when x has a particular value. If we add the same number, like 5, to both $3x-5$ and to 16, the results are still equal for the same variable value. This means the statement $3x=21$ must be true for the same value of x."

RESPONSIVE STRATEGY

Display the following sentence frames for all to see: "____ and ____ are both true because….", and "I noticed ____ and ____ are not equivalent because…." Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about equivalent equations.

Discussion Supports (MLR8)

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students a few minutes of quiet time to think about the equivalent equations and the non-equivalent equations.
  - Then, give students time to *Take Turns* sharing their explanations with their partner.

- Remind students that when one student explains, the partner's job is to listen and make sure that they agree and that the explanation makes sense. If they don't agree, the partners discuss until they come to an agreement.

  - Facilitate the *Collect and Display* routine. As students take turns sharing explanations, circulate and listen for good examples of student talk that align with the guidance in Step 1 (about explanations vs. descriptions of steps). Scribe and display a few examples of student talk in a place where everyone can see to reinforce that guidance and create additional examples in real time. The collected student language may be helpful as a reference for the next activity.

Monitoring Tip: As students discuss their thinking, listen for explanations that are less clear and probe students to refine their responses in preparation for sharing with the class later.

***Advancing Student Thinking*:** Students may have trouble seeing why some equations are not equivalent (particularly the second item, which contains a common error). Encourage these students to choose one pair of equations, solve one equation, and then substitute the solution into the other equation to see what goes wrong.

Student Task Statement

Here are some pairs of equations. Partners take turns, row by row, being an explainer and a note taker.

One partner listens and takes notes while the other partner explains why:

- For the equivalent equations, if x is a number that makes the first equation true, then it also makes the second equation true.
- For the non-equivalent equations, the second equation is no longer true for a value of x that makes the first equation true.

Then, switch roles for the following row.

	Equivalent equations	Notes	Non-equivalent equations	Notes
1.	$16 = 4(9 - x)$ $16 = 36 - 4x$		$9x = 5x + 4$ $14x = 4$	
2.	$5x = 24 + 2x$ $3x = 24$		$\frac{1}{2}x - 8 = 9$ $x - 8 = 18$	
3.	$-3(2x + 9) = 12$ $2x + 9 = -4$		$6x - 6 = 3x$ $x - 1 = 3x$	
4.	$5x = 3 - x$ $5x = -x + 3$		$-11(x - 2) = 8$ $x - 2 = 8 + 11$	

Step 3

Invite previously identified students to share their explanations on at least a couple of pairs of equations from each column. If not already clear from students' explanations, emphasize that:

- If two expressions are equal, then performing the same addition (or subtraction) or multiplication (or division) to both expressions maintains the equality.
- Applying the distributive, commutative, and associative properties (of multiplication or addition) to an expression doesn't change its value; therefore, doing so to one side of a true equation doesn't change the equality.

Explain to students that next they will look at some examples where the moves made to write equivalent equations appear to be acceptable but the resulting equations turn out to be false statements.

RESPONSIVE STRATEGIES

Use color and annotations to illustrate student thinking. As students share their reasoning about the pairs of equations, scribe their thinking on a visible display. Invite students to compare moves described that produce equations with the same solutions, to those that create equations with different solutions.

Supports accessibility for: Visual-spatial processing; Conceptual processing

DO THE MATH	PLANNING NOTES

Activity 2: It Doesn't Work! (*10 minutes*)

Instructional Routines: Round Robin; Collect and Display (MLR2); Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-REI.1	

So far, students have seen only equations that have a solution. For these equations, performing acceptable moves always led to equivalent equations that have the same solution. In this activity, students encounter an example where the given equation has no solutions, and performing the familiar moves leads to an untrue statement.

Prior to this point, students have added, subtracted, multiplied, and divided a number from both sides of an equation. They have also added a variable expression to (or subtracted a variable expression from) an equation. They recognize these moves as allowable for solving equations. Here, students examine what happens when each side of an equation is divided by a variable expression and make sense of why doing so leads to a false statement.

Step 1

- Ask students to arrange themselves into small groups or use visibly random grouping.
- Give groups 1–2 minutes of quiet time to analyze the first set of equations and then use the *Round Robin* routine within their group to brainstorm why Noah's work results in a false statement.

RESPONSIVE STRATEGY

Give students additional time to make sure everyone in their group can explain their analysis of Noah's moves. Invite groups to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking, and will improve the quality of explanations shared during the whole-class discussion.

Discussion Supports (MLR8)

Step 2

- Invite students to share their explanations as to why Noah ended up with $6 = 1$. Continue the *Collect and Display* routine by referring to (and building on, where relevant) the collected language from the previous activity. Students are likely to say that the moves Noah made were allowable and can be explained, but they may struggle to say why they end up with a false statement.
- Draw students' attention to the third line of Noah's work. Ask them to interpret the expression on each side of the equal sign: $x + 6$ can be interpreted as 6 more than a number, and $x + 1$ as 1 more than that same number. Then, ask them to try to find a value of x that could make that equation true. (There is none!)
- Highlight that it's not possible for 6 more than some number, no matter what that number is, to be equal to 1 more than that number. If no value of x could make the third equation true, the same could be said about the original equation—it had no solution. So even though Noah performed acceptable moves, the final equation is a false equation. Because all of these equations are equivalent, this means the original equation is also a false equation. It has no solutions because no value of x could make the equation true.
- Repeat the process to analyze the second set of equations.

***Advancing Student Thinking*:** Some students may point to a step that is valid and mistakenly identify it as an error. For instance, in the first set of steps, they may object to replacing $1–3x$ with $–3x + 1$, thinking that it should be rearranged to $3x–1$. Push their reasoning with a simpler example. Ask, for instance, if $7–4$ is equivalent to $4–7$. Remind students that we can think of $7–4$ as $+7 + (-4)$ and then apply the commutative property of addition to get $-4 + 7$.

If students hypothesize about two equations being equivalent but are not sure how to check if it's actually the case, suggest that a good way to check is by finding the solution to one equation, then checking whether that value is also a solution to the second equation.

Student Task Statement

Noah is having trouble solving two equations. In each case, he took steps that he thought were acceptable but ended up with statements that are clearly not true.

Analyze Noah's work on each equation and the moves he made. Were they acceptable moves? Why do you think he ended up with a false equation?

Discuss your observations with your group and be prepared to share your conclusions. If you get stuck, consider solving each equation.

1.
$$\begin{aligned} x+6 &= 4x+1-3x && \text{original equation}\\ x+6 &= 4x-3x+1 && \text{apply the commutative property}\\ x+6 &= x+1 && \text{combine like terms}\\ 6 &= 1 && \text{subtract x from each side}\end{aligned}$$

2.
$$\begin{aligned} 2(5+x)-1 &= 3x+9 && \text{original equation}\\ 10+2x-1 &= 3x+9 && \text{apply the distributive property}\\ 2x-1 &= 3x-1 && \text{subtract 10 from each side}\\ 2x &= 3x && \text{add 1 to each side}\\ 2 &= 3 && \text{divide each side by x}\end{aligned}$$

Step 3

- Invite students to share what they thought was the problem with Noah's work. They are likely to say that Noah seems to have performed allowable moves and did them correctly. Then, draw students' attention to the second-to-last step: $2x = 3x$. Ask students:
  - "Earlier, when looking at $x + 6 = x + 1$, we reasoned that there's no value of x that could make this equation true. Is there a value of x that could make $2x = 3x$ true?" (Yes, 0.)
  - "If there is a value that can make $2x = 3x$ true, what do we know about the original equation? Is it also true?" (Yes, they are all equivalent equations.) "What is its solution?" (0)
  - "So the equations are true up through the third step. The last step is where we have a false statement, after Noah shows division by x. If x is 0, what might be problem with dividing by x?" (If x is 0, then we are dividing both sides by 0, which gives an undefined result.)
- Explain that dividing by the variable in the equation is not done because if the solution happens to be 0, it could lead students to think that there is no solution while in fact there is. (The solution is the number 0.)
- Revisit the lists of acceptable and unacceptable moves compiled in earlier activities. Add "dividing by the variable" and "dividing by 0" to the list of unacceptable moves.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students learn to explain why the acceptable moves to solve equations keep the solutions the same, and to show that other moves don't. In addition, they see that seemingly acceptable moves cannot help to find a solution if the equation had no solution in the first place, and that dividing by a variable can eliminate solutions. Center a discussion around the four equations here, which are examples of each.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through the questions with a partner, and what will be prioritized in the full class discussion.

Display the following sets of equations and questions for all to see. Tell students that each set represents an original equation and the first step taken to solve it.

a. $$\begin{aligned} 5(x-3) &= 5 \\ x-3 &= 1 \end{aligned}$$

b. $$\begin{aligned} 5x-3 &= 5 \\ 5x &= 2 \end{aligned}$$

c. $$\begin{aligned} 5(x-3) &= 5x \\ x-3 &= x \end{aligned}$$

d. $$\begin{aligned} (5-3)x &= 5x \\ 5-3 &= 5 \end{aligned}$$

PLANNING NOTES

For each set of equations, ask students:

- "What move was made to the original equation to obtain the second equation?"
- "Is the solution to the second equation the same as the solution to the original equation? Why does it stay the same or why does it change?"

Make sure students see the following for each equation listed below:

a. The move made is dividing each side by 5. The solution is the same $(x=4)$ because dividing both sides by the same number keeps the two sides equal.

b. The move made is adding 3 to the left side and -3 to the right. The solution to the first equation $(x=\frac{8}{5})$ no longer makes the second equation true because the two sides of the equation are no longer equal.

c. The move made is dividing each side by 5. It is a valid move, but because the original equation does not have a solution (5 times a number, $5x$, cannot be equal to 5 times a smaller number, $5(x-3)$), the second equation also does not have a solution (3 less than a number, $x-3$, cannot be equal to that number, x).

d. The move made is dividing each side by x. The second equation doesn't have a solution even though the original solution does. The move is problematic because it would eliminate the variable and make us miss the solution, namely $x=0$.

Student Lesson Summary and Glossary

When solving an equation, sometimes we end up with a false equation instead of a solution. Let's look at two examples.

Example 1: $4(x+1)=4x$

Here are two attempts to solve it.

$$\begin{aligned} 4(x+1) &= 4x && \text{original equation} \\ x+1 &= x && \text{divide each side by 4} \\ 1 &= 0 && \text{subtract x from each side} \end{aligned}$$

$$\begin{aligned} 4(x+1) &= 4x && \text{original equation} \\ 4x+4 &= 4x && \text{apply the distributive prop} \\ 4 &= 0 && \text{subtract 4x from each side} \end{aligned}$$

Each attempt shows acceptable moves, but the final equation is a false statement. Why is that?

When solving an equation, we usually start by assuming that there is at least one value that makes the equation true. The equation $4(x+1)=4x$ can be interpreted as: 4 groups of $(x+1)$ are equal to 4 groups of x. There are no values of x that can make this true.

For instance, if $x=10$, then $x+1=11$. It's not possible that 4 times 11 is equal to 4 times 10. Likewise, 1.5 is 1 more than 0.5, but 4 groups of 1.5 cannot be equal to 4 groups of 0.5.

Because of this, the moves made to solve the equation would not lead to a solution. The equation $4(x+1)=4$ has no solutions.

Example 2: $2x - 5 = \frac{x-20}{4}$

$$\begin{array}{rcll} 2x-5 & = & \frac{x-20}{4} & \text{original equation} \\ 8x-20 & = & x-20 & \text{multiply each side by 4} \\ 8x & = & x & \text{add 20 to each side} \\ 8 & = & 1 & \text{divide each side by x} \end{array}$$

Each step in the process seems acceptable, but the last equation is a false statement.

It is not easy to tell from the original equation whether it has a solution, but if we look at the equivalent equation $8x = x$, we can see that 0 could be a solution. When x is 0, the equation is $0 = 0$, which is a true statement. What is going on here?

The last move in the solving process was division by x. Because 0 could be the value of x and dividing by 0 gives an undefined number, we don't usually divide by the variable we're solving for. Doing this might make us miss a solution, namely $x = 0$.

Cool-down: If This, Then That (*5 minutes*)

Addressing: NC.M1.A-REI.1

Cool-down Guidance: Address student misconceptions about dividing by a variable at the beginning of the next lesson. If students struggle with division in the first part of the cool-down, look for moments to emphasize the procedure of doing the same thing to both sides in Lessons 7 and 8.

Cool-down

1. The equation $4(x–2) = 100$ is a true equation for a particular value of x. Explain why $2(x–2) = 50$ is also true for the same value of x.
2. To solve the equation $7.5d = 2.5d$, Lin divides each side by $2.5d$, and Elena subtracts $2.5d$ from each side.
   a. Will both moves lead to the solution? Explain your reasoning.
   b. What is the solution?

Student Reflection:

Today my participation was (circle one): *high* *medium* *low* because ______________________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In an upcoming lesson, students will create and solve multi-step equations. What do you notice in their work from today's lesson that you might leverage in that future lesson?

Practice Problems

1. Match each equation from the left column with an equivalent equation from the right column. Some of the answer choices are not used.

a. $3x + 6 = 4x + 7$	1. $9x = 4x + 7$
b. $3(x + 6) = 4x + 7$	2. $3x + 18 = 4x + 7$
c. $4x + 3x = 7 - 6$	3. $3x = 4x + 7$
	4. $3x - 1 = 4x$
	5. $7x = 1$

2. Mai says that equations A and B have the same solution.

- Equation A: $-3(x + 7) = 24$
- Equation B: $x + 7 = -8$

(*continued*)

Which statement explains why this is **true**?

a. Adding 3 to both sides of equation A gives $x+7=-8$.
b. Applying the distributive property to equation A gives $x+7=-8$.
c. Subtracting 3 from both sides of equation A gives $x+7=-8$.
d. Dividing both sides of equation A by -3 gives $x+7=-8$.

3. Is 0 a solution to $2x+10=4x+10$? Explain or show your reasoning.

4. Kiran says that a solution to the equation $x+4=20$ must also be a solution to the equation $5(x+4)=100$. Write a convincing explanation as to why this is true.

5. The depth of two lakes is measured at multiple spots. For the first lake, the mean depth is about 45 feet with a standard deviation of 8 feet. For the second lake, the mean depth is about 60 feet with a standard deviation of 27 feet.

   Noah says the second lake is generally deeper than the first lake. Do you agree with Noah?

6. Which equation is equivalent to the equation $5x+30=45$?

   a. $35x=45$
   b. $5x=75$
   c. $5(x+30)=45$
   d. $5(x+6)=45$

   (From Unit 2, Lesson 5)

7. Select **all** the equations that have the same solution as $2x–5=15$.

   a. $2x=10$
   b. $2x=20$
   c. $2(x–5)=15$
   d. $2x–20=0$
   e. $4x–10=30$
   f. $15=5–2x$

   (From Unit 2, Lesson 5)

8. Diego's age d is 5 more than 2 times his sister's age s. This situation is represented by the equation $d=2s+5$.

   Which equation is equivalent to the equation $d=2s+5$?

   a. $d=2(s+5)$
   b. $d-5=2s$
   c. $d-2=s+5$
   d. $\frac{d}{2}=s+5$

   (From Unit 2, Lesson 5)

9. What effect does eliminating the lowest value, -6, from the data set have on the mean and median?

   -6, 3, 3, 3, 3, 5, 6, 6, 8, 10

   (From Unit 1)

10. For each equation, decide if $x=12$ is a solution.

   a. $6=\frac{1}{2}x$

   b. $18=2x$

   (Addressing NC.6.EE.5)

Lesson 7: Creating and Solving Equations (Part One)[1]

PREPARATION

Lesson Goal	Learning Targets
• Solve equations fluently and understand that different approaches can lead to the same correct answer.	• I can solve equations. • I understand why equations can be solved in multiple ways.

Lesson Narrative

This lesson is focused on fluency with solving equations. Students will build off of previous lessons by applying the rules for solving equations, including those with rational numbers, variables on both sides of the equal sign, and requiring multiple steps. Students will have the opportunity to work with a partner to gain appreciation for the fact that equations can be solved using different moves: for example, distributing or dividing. As they examine these moves, students are asked to justify why they result in equivalent equations. While fluently solving equations is an expectation in grade 8, this lesson provides additional practice manipulating more complex equations.

What is the main purpose of this lesson? What is the one thing you want your students to take away from the lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.5: Use substitution to determine whether a given number in a specified set makes an equation true. **NC.7.EE.4:** Use variables to represent quantities to solve real-world or mathematical problems. **a**. Construct equations to solve problems by reasoning about the quantities. • Fluently solve multistep equations with the variable on one side, including those generated by word problems. (*continued*)	**NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

• Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. • Interpret the solution in context. **NC.8.EE.7**: Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.	

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
  - Trading Moves equation cards (print 1 copy per every 2 students)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L7 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.4

The purpose of this bridge is to provide students the opportunity to solve one-step equations with rational numbers before they practice solving multi-step equations later in this lesson. Students who struggled with questions 2 and 4 on the Check Your Readiness may benefit from this additional practice.

Student Task Statement

Find a solution to each equation:

1. $-3+x=8$
2. $-10=12+x$
3. $\frac{2}{3}x=6$
4. $4x=-26$

DO THE MATH	PLANNING NOTES

Warm-up: Is It a Solution? (*10 minutes*)

Building On: NC.6.EE.5	**Building Towards:** NC.M1.A-REI.1

The purpose of this activity is to give students an opportunity to practice checking whether a particular value is a solution to an equation, and to recall properties of operations and equality that preserve the solution set to an equation.

This will be useful when students consider when and why equations have the same solution in the later lessons.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give approximately 3 minutes of individual work time.

> **Monitoring Tip:** While students are working on the problems individually, walk the room to monitor for students using different strategies: solving the equations, manipulating the expressions, substituting the value in only one expression and reasoning about the second expression, and identifying properties by name such as commutative, distributive, and associative.

- Have students compare their answers with a partner.

Student Task Statement

For each pair of equations, decide whether the given value of x is a solution to one or both equations:

1. Is $x = 2$ a solution to:
   a. $x(2+3) = 10$
   b. $2x + 3x = 10$
2. $x = 3$ a solution to:
   a. $x - 4 = 1$
   b. $4 - x = 1$
3. $x = 5$ a solution to:
   a. $4x - 5 = 2x + 5$
   b. $4x + 2x = 5 + 5$

Step 2

- Facilitate a whole-class discussion by asking previously identified students to share the different strategies that were used. Have students visually share their work so the rest of the class can easily follow along.

DO THE MATH	PLANNING NOTES

Activity 1: Trading Moves (*20 minutes*)

Instructional Routines: Take Turns; Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.8.EE.7	**Addressing:** NC.M1.A-REI.1; NC.M1.A-REI.3

The goal of this activity is for students to build fluency in solving equations with variables on each side. Students describe each step in their solution process to a partner and justify how each of their changes maintains the equality of the two expressions.

If time is a concern, give each group two cards rather than all four and have them only doing the trading steps portion of the activity, but make sure that all four cards are distributed throughout the class. Make sure each problem is discussed in the final whole-group discussion. Alternatively, if there is additional time, extend the activity by selecting more problems for students to solve with their partners.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Instruct the class that they will receive four cards with problems on them, and that they will use the *Take Turns* routine to create a solution to the problems.
- Review the directions with the whole class. If needed, help students understand how they are expected to solve the first two problems using the trading process by demonstrating with a student volunteer and a sample equation.
  - Emphasize that the "why" justification should include how their step maintains the equality of the equation. Remind students to push each other to explain how their step guarantees that the equation is still balanced as they are working. For example, a student might say they are combining two terms on one side of the equation, which maintains the equality as the value of that side does not change, only the appearance.
- Students have approximately 10–12 minutes to complete the activity.

RESPONSIVE STRATEGY

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed the initial set, or create an additional set of cards with equations that are more accessible.

Supports accessibility for: Conceptual processing

M **Monitoring Tip:** While students are working on the task, look for groups solving problems in different, but efficient, ways. For example, one group may distribute the 1/2 on the left side in problem 2 while another may multiply each side of the equation by 2 in order to rewrite the equation with fewer terms on each side.

***Advancing Student Thinking*:** This activity may not have many misconceptions but rather many errors. For example, students may forget to apply the distributive property, incorrectly add integers, or struggle with rational numbers. Capture informal data to support teacher-led small-group work during the Checkpoint Lessons.

Student Task Statement

Your teacher will give you four cards, each with an equation.

1. With your partner, select a card and choose who will take the first turn.
2. During your turn, decide what the next move to solve the equation should be, explain your choice to your partner, and then write it down once you both agree. Switch roles for the next move. This continues until the equation is solved.
3. Choose a second equation to solve in the same way, trading the card back and forth after each move.
4. For the last two equations, choose one each to solve and then trade with your partner when you finish to check each other's work.

Are You Ready For More?

Your class is asked to solve this equation.

$$\frac{1}{2}(x+5)=\frac{1}{3}(2x-4)+8$$

Your friend freezes up a bit when they see fractions. Is there an equivalent equation your friend could use that would get the same solution for x but would eliminate the fractions before distributing?

Step 2

- After the students have completed the task, facilitate a whole-group discussion. The purpose of this discussion is for the class to see different, successful ways of solving the same equation and to gain confidence in their ability to solve equations efficiently, even if their strategy differs from that of their peers. Record and display the student thinking that emerges during the discussion to help the class follow what is being said. To highlight some of the differences in solution paths, ask:
  - "Did your partner ever make a move different than the one you expected them to? Describe it."
  - "How did you check if your partner's solution was correct?"
  - "What's an arithmetic error you made but then caught when you checked your work?"

RESPONSIVE STRATEGY

Use this routine to support students in producing statements about common errors in problem solving. Use the example offered, (the second line has __ instead of __ for problem 2), and provide sentence frames to support the discussion. For example, "The error this student made was… and I believe this happened because…." and "A different solution path could be…." Restate or revoice student language to demonstrate use of correct mathematical language to describe each move (e.g., "distribute the ," "combine like terms," etc.), and include mathematical reasoning (e.g., "…because this maintains the equality"). Clarify explanations that detail differences in problem-solving strategies rather than errors, to help students see differences in solution paths. This will help students describe differences in solution paths and justify each step.

Discussion Supports (MLR8)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students explored a variety of strategies for solving equations. The goal of the debrief is to help them internalize some new strategies and to remember some common errors that came up in the course of the lesson.

	PLANNING NOTES
Choose whether students should first have an opportunity to reflect in their workbooks or talk through these questions with a partner. • Thinking about the equations you solved and worked with today, what are some errors you made or observed? What are some strategies you saw other people use that were different from the ones you used? • Allow students to share their responses and consider creating a permanent display showing: – different approaches for different structures of equations – types of errors to look out for	

Student Lesson Summary and Glossary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.[2]

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful ways to start are:

- divide each side by $\frac{4}{5}$
- apply the distributive property

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5}\cdot 27$ will be hard, because 27 isn't divisible by 5. But $16 \div \frac{4}{5}$ gives us $16\cdot\frac{5}{4}$, and 16 is divisible by 4. Dividing each side by $\frac{4}{5}$ gives:

$$\begin{aligned}
\tfrac{4}{5}(x+27) &= 16 \\
\tfrac{5}{4}\cdot\tfrac{4}{5}(x+27)) &= 16\cdot\tfrac{5}{4} && \text{Multiplying each side of the equation by the same number } (\tfrac{5}{4}) \text{ keeps the two expressions equal} \\
x+27 &= 20 && \text{Simplifying each side of the equation preserves the equality} \\
x &= -7 && \text{Subtracting each side of an equation by the same number preserves the equality}
\end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned}
100(x+0.06) &= 21 \\
100x+6 &= 21 && \text{Using the distributive property on } 100(x+0.06) \text{ creates an equivalent expression, which must also equal 21} \\
100x &= 15 && \text{Subtracting the same number from each side of an equation preserves the equality} \\
x &= \tfrac{15}{100} && \text{Dividing each side of an equation by the same number preserves the equality}
\end{aligned}$$

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[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/teachers/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017-2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Cool-down: Solve and Justify (*5 minutes*)

Addressing: NC.M1.A-REI.1

Cool-down Guidance: More Chances
Students will continue to build their fluency with respect to solving equations in the next lesson and throughout the unit.

Cool-down

Solve both equations and justify why the equality is preserved after each move.

1. $\frac{1}{2}(x+6)=2(5-x)$
2. $3x-10=2x+12$

Student Reflection: After today's lesson I feel _______________. Explain why.

a. very strong
b. I am getting better
c. frustrated with math

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What makes someone good at math? Are there ways that you make assumptions about which of your students are good at math?

Practice Problems

1. Solve each equation and show your work.[3]

   a. $2b + 8 - 5b + 3 = -13 + 8b - 5$
   b. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

2. Clare was solving an equation, but when she checked her answer she saw her solution was incorrect. It turns out Clare's work has three mistakes!

$$\begin{aligned} 8(2+7x) &= 3x-(2+5x) \\ 10+56x &= 3x-2+5x \\ 10+56x &= 8x-2 \\ 10 &= 64x-2 \\ 12 &= 64x \\ \tfrac{3}{16} &= x \end{aligned}$$

   a. Find all three mistakes.

   b. Find the solution to the equation.

3. Consider the equation $3x + 4 = 8x - 16$. Solve for x using the given starting point.[4]

Group 1	Group 2	Group 3	Group 4
Subtract $3x$ *from both sides*	*Subtract* 4 *from both sides*	*Subtract* $8x$ *from both sides*	*Add* 16 *to both sides*

4. Determine which of the following equations have the same solution set by recognizing properties rather than solving. Then solve only those that you determine have the same solution set.[5]

   a. $2x + 3 = 13 - 5x$
   b. $6 + 4x = -10 + 26$
   c. $6x + 9 = \frac{13}{5} - x$
   d. $0.6 + 0.4x = -x + 2.6$
   e. $3(2x + 3) = \frac{13}{5} - x$
   f. $4x = -10x + 20$
   g. $15(2x + 3) = 13 - 5x$
   h. $15(2x + 3) + 97 = 110 - 5x$

5. The environmental science club is printing T-shirts for its 15 members. The printing company charges a certain amount for each shirt plus a setup fee of $20.

   If the T-shirt order costs a total of $162.50, how much does the company charge for each shirt?

   (From Unit 2, Lesson 4)

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[4] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[5] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

6. A group of 280 elementary school students and 40 adults are going on a field trip. They are planning to use two different types of buses to get to the destination. The first type of bus holds 50 people, and the second type of bus holds 56 people. Andre says that three of the first type of bus and three of the second type of bus will hold all of the students and adults going on the field trip. Is Andre correct? Explain your reasoning.

   (From Unit 2, Lesson 4)

7. Which of the following equations have the same solution set? Give reasons for your answers that do not depend on solving the equations.[6]

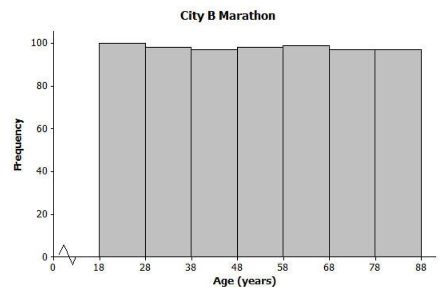
   a. $x - 5 = 3x + 7$
   b. $3x - 6 = 7x + 8$
   c. $15x - 9 = 6x + 24$
   d. $6x - 16 = 14x + 12$
   e. $9x + 21 = 3x - 15$
   f. $-0.05 + \frac{x}{100} = \frac{3x}{100} + 0.07$

   (From Unit 2, Lesson 6)

8. A large city, which we will call City A, holds a marathon. Suppose that the ages of the participants in the marathon that took place in City A were summarized in the histogram below.[7]

   a. Make an estimate of the mean age of the participants in the City A marathon.

   A smaller city, City B, also held a marathon. However, City B restricts the number of people of each age category who can take part to 100 people. The ages of the participants are summarized in the histogram below.



   b. Approximately what was the mean age of the participants in the City B marathon? Approximately what was the standard deviation of the ages?

   c. Explain why the standard deviation of the ages in the City B marathon is greater than the standard deviation of the ages for the City A marathon.

   (From Unit 1)

9. Find the value of each variable.[8]

   a. $a \cdot 3 = -30$

   b. $-9 \cdot b = 45$

   (Addressing NC.7.EE.4)

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Lesson 8: Creating and Solving Equations (Part Two)[1]

PREPARATION

Lesson Goals	Learning Targets
• Write and solve equations from story problems. • Solve equations fluently.	• I can solve equations. • I can solve story problems by writing and solving an equation.

Lesson Narrative

This lesson continues to focus on building fluency with respect to solving equations. As in the previous lesson, students will strengthen their skill of solving equations, including those with variables on both sides of the equal sign, rational numbers, and those that require multiple steps. An additional focus is on creating equations from varying contexts.

What are the skills that students should take away after this lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.3: Apply the properties of operations to generate equivalent expressions without exponents. **NC.8.EE.7:** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.	**NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L8 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.3

The purpose of this bridge is for students to have another opportunity to practice combining like terms. Later in this lesson, students will need to solve multi-step equations with variables on each side of the equal sign.

Student Task Statement

Priya and Elena are both trying to write an expression with fewer terms that is equivalent to $7a + 5b - 3a + 4b$.

- Priya thinks $10a + 1b$ is equivalent to the original expression.
- Elena thinks $4a + 9b$ is equivalent to the original expression.

Who do you agree with and why?[2]

DO THE MATH	PLANNING NOTES

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Warm-up: Reflection (*10 minutes*)

Building On: NC.8.EE.7

The goal of this activity is for students to create a linear equation, solve the equation, and then reflect on which values of the variable would be reasonable answers.

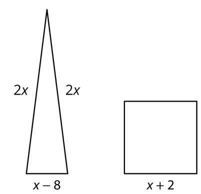
Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students 2 minutes quiet think time and then 2 minutes to discuss their solutions with a partner.
- Instruct groups to explain to each other how they came up with expressions and an equation to represent the situation.

Student Task Statement

The triangle and the square have equal perimeters.

1. Find the value of x.
2. What are the perimeters of each of the figures?



Step 2

Ask groups to share their strategies for solving the question. Consider asking some of the following questions:

- "What expression represents the perimeter of the triangle? The perimeter of the square?" (The expression for perimeter of the triangle is $5x - 8$, and the perimeter of the square is $4(x+2)$.)
- "What was your strategy in making an equation?" (If both perimeters are the same, we can say their expressions are equal.)
- "What does x mean in the situation?" (It means an unknown value. None of the sides or perimeter is represented by x, so we cannot say it represents a specific length in the figures.)
- "Looking at the figures, are there any values that x could not be? Explain your reasoning." (Since the triangles have sides that are $2x$, x cannot be 0 or a negative value. Triangles cannot have sides with 0 or negative side lengths. Since the third side is $x - 8$, we can use this same reasoning to realize that x must actually be greater than 8.)
- "How does this information help when solving?" (If I make a mistake in my solution and get a value of x that is less than or equal to 8, then I know immediately that my answer is not reasonable and I can try to find my error.)

DO THE MATH

PLANNING NOTES

Activity 1: Squirrels and Chipmunks and More Equations (*20 minutes*)

Instructional Routines: Three Reads (MLR6); Round Robin	
Building On: NC.8.EE.7	**Addressing:** NC.M1.A-CED.1; NC.M1.A-REI.3

The purpose of this activity is to have students think about how information can be transformed into an equation in order to solve for an unknown. The activity offers four word problems.

This is the first time in the course that students will participate in a *Three Reads* routine.

THREE READS	***What Is This Routine?*** A word problem is read three times, with a different question posed with each read: (1) What is this situation about?; (2) What can be counted or measured in this situation?; (3) How might we approach this problem, or what is the first thing you will do to get started? ***Why This Routine?*** *Three Reads* (MLR6) gives students a chance to use everyday language to help each other make sense of the context—and the language—of a word problem before jumping down a solution path. Use this routine to ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the general structure of quantitative situations and on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with peers through mathematical conversation.

Step 1

- Use the *Three Reads* routine to get students started on problem 1.
  - First Read: Without displaying the problem, read the problem aloud to the class.
    - Ask students: "What is this situation about? What is going on here?" Let students know the focus is just on the situation, not on the numbers. (For example, students might say, "it's about squirrels hiding acorns" or "it's about a chipmunk and a squirrel and digging holes.")
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words; visuals often help (for example, a picture of a squirrel burying nuts).
  - Second Read: Display the problem and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Again, spend less than a minute scribing student responses. Encourage students to identify quantities that are named in the problem explicitly, and any quantities that may be implicit. For each quantity (for example, "three acorns"), ask students to add details (for example, "three acorns hidden by the chipmunk").
  - Third Read: Invite students to read the problem again to themselves, or ask another student volunteer to read it aloud.
    - Ask: "How might we approach the question being asked? What is the first thing you will do?"
    - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points. Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.
- Offer students the opportunity to work independently or with a partner to solve the problem.

Student Task Statement

1. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid three acorns in each of the holes it dug. The squirrel hid four acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed four fewer holes. How many acorns did the chipmunk hide?[3]

Step 2

- Have students arrange themselves in small groups or use visibly random grouping.
- Give students 3 minutes of individual work time to create equations for problems 2 and 3, and then use the *Round Robin* routine to structure a discussion of the equations they created.
- Once students have come to agreement on the equations, ask them to solve the equations independently and then compare their solutions within their groups.
- Depending on time constraints, you may have all students complete all three problems or assign a different problem to each group.

Student Task Statement

2. Mai was hired part time at Shoe Box shoe store as a sales clerk. She earns $84.00 a day plus $3.00 for every pair of shoes she sells. She wants to earn $114 a day. Write an equation and solve to find how many pairs of shoes, p, she must sell to make her goal.
3. Coach Hicks and the basketball team bought bottles of water to pass out to those unhoused in their community. The water was on sale for $4.30 per pack of 12. They also needed to pay $3.40 total for reusable bags. They have $55.00 to spend. Write an equation and solve to determine how many packs of water the team can buy.
4. A zookeeper realized that an alligator in the zoo is now four times the age that it was 15 years ago. How old is the alligator now?

Are You Ready For More?

The movie was so boring, Andre walked out of the theater after seeing only a quarter of it. Fifteen minutes later, Mae walked out after seeing a third of it. How long was the movie?[4]

Step 3

- Facilitate a whole-class discussion asking about the types of information/phrases in word problems that are important for writing equations. Consider asking some of the following questions to further the discussion:
  - "What words/phrases in the problems helped you to write equations?"
  - "What information did you find helpful to writing your equations?"
  - "What steps did you take to write your equations?"
  - "What steps did you take to solve your equations?"
  - "What other strategies or steps did you use in solving the equations?"

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[3] Adapted from Illustrativemathematics.org
[4] Adapted from AchievetheCore.org

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson debrief is to have students create their own word problems. In doing so, they will consider what types of quantities can be variables, and what operations will be relevant to describe their chosen scenarios.

	PLANNING NOTES
• Ask students to write their own word problem that would require creating and solving an equation using a subject of their choice. Remind students to pay attention to the words and phrases being used in problems in which they must write equations. • Consider a way to collect the word problems to be used as practice problems during future lessons or in a station in the next lesson. • Highlight a few word problems to display and ask students to choose one to solve, if time allows.	

Student Lesson Summary and Glossary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next 6 days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

Before writing an equation, it's important to understand the situation. This might require reading the problem more than once.

Once we understand that this is a problem about running a certain distance each day, we look at the quantities. We see that there is information about the number of miles for both one day (4 miles) and for the total week (22 miles). We see that the same distance was run for each of the 6 days following Monday. We also see that the number of days is a quantity we have to pay attention to. Finally, we want to understand what the question is asking. In this case, the question is, "How many miles did she run on each of the 6 days?" This tells us that the distance Clare runs each of the 6 days, in miles, is unknown. We can represent this quantity by a variable, m.

Since m is the distance Clare runs on each of the 6 days, we can represent the total amount she runs on those days as $6m$. We can then add the 4 miles she runs on Monday to that total, making $4+6m$ the distance Clare ran throughout the week. We also know Clare ran a total of 22 miles throughout the week. This gives the equation $4+6m=22$.

Solving the equation will tell us the distance Clare ran on each of the 6 days.[5]

$$\begin{aligned} 4+6m &= 22 \\ 6m &= 18 \\ m &= 3 \end{aligned}$$

So Clare ran 3 miles on each of the 6 days.

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Cool-down: Declining Account (*5 minutes*)

Addressing: NC.M1.A-CED.1

Cool-down Guidance: Press Pause
Provide students who are not able to set up an equation for the cool-down scenario with additional support during small group teacher-led instruction during the Checkpoint lessons.

Cool-down[6]

A checking account is set up with an initial balance of \$4800, and \$400 is removed from the account each month for rent (no other transactions occur on the account). Write an equation whose solution is the number of months, m, it takes for the account balance to reach \$2000.

Student Reflection:

1. What have you learned about writing and solving equations?
2. What are you really good at?
3. What should you keep working towards?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

[6] Adapted from illustrativemathematics.org

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a recent time from class when your students were confused. What did you do to support them in reasoning about their confusion together as a community of learners?

Practice Problems

1. Lin and Noah are solving the equation $7(x+2) = 91$. Lin starts by using the distributive property. Noah starts by dividing each side by 7.
   a. Show what Lin's and Noah's full solution methods might look like.
   b. What is the same and what is different about their methods?[7]
2. A family of six is going to the fair. They have a coupon for $1.50 off each ticket. If they pay $46.50 for all their tickets, how much does a ticket cost without the coupon? Write an equation to find the ticket cost without a coupon.[8]
3. Here are two stories:[9]
   - The initial freshman class at a college is 10% smaller than last year's class. But then during the first week of classes, 20 more students enroll. There are then 830 students in the freshman class.
   - A store reduces the price of a computer by $20. Then during a 10% off sale, a customer pays $830.

   Here are two equations:
   - $0.9x + 20 = 830$
   - $0.9(x-20) = 830$

   Questions:
   a. Decide which equation represents each story.
   b. Explain why one equation has parentheses and the other doesn't.
   c. Solve each equation and explain what the solution means in the situation.
4. Solve the equations.[10]
   a. $2(x-3) = 14$
   b. $\frac{5}{7}(x-9) = 25$
   c. $\frac{1}{6}(x+6) = 11$
   d. $-5(x-1) = 40$
5. Write a real-world problem that could be represented by the equation $2(3x+1) = 14$ and solve for x.
6. Diego was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been?[11] Identify at least five equations that have a solution of $x = 3$.
7. Lin ordered 250 pens and 250 pencils to sell for a theatre club fundraiser. The pens cost 11 cents more than the pencils. If Lin's total order costs $42.50, find the cost of each pen and pencil.[12]

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[9] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).
[10] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).
[11] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[12] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

8. Solve the equation below.[13] Identify the moves that are easiest and the moves that are most difficult for you.

$$\frac{2(x+4)}{3} = 12$$

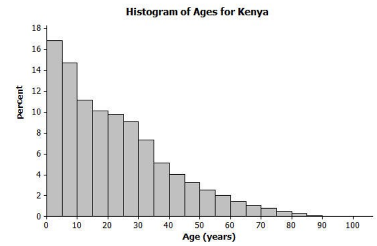
9. Review the histograms.[14]

   a. Using the histograms of the population distributions of the United States and Kenya in 2010, approximately what percent of the people in the United States were between 15 and 50 years old? Approximately what percent of the people in Kenya were between 15 and 50 years old?

   b. What five-year interval of ages represented in the 2010 histogram of the United States age distribution has the most people?

   c. Why is the mean age greater than the median age for people in Kenya?

   (From Unit 1)



[13] Adapted from Math 1, Module 4 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)
[14] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Lessons 9 & 10: Checkpoint

PREPARATION

Lesson Goals	Learning Targets
• Learn and grow mathematically in course-level content. • Communicate and address mathematical areas of strength and areas of growth.	• I can share what I know mathematically. • I can continue to grow as a mathematician and challenge myself.

Lesson Narrative

This is a Checkpoint day. Checkpoint days consist of two lessons (one full block) and are structured as four 20-minute stations that students rotate between. There are a total of seven stations students can engage with. Since students will not be able to participate in all seven stations, please note that Station A (Unit 3 Check Your Readiness) is required for all students.

A. Unit 3 Check Your Readiness (*Required*)
B. Teacher-led Small-group Instruction
C. Let's Go to the Carnival
D. Weekend Earnings
E. Micro-modeling
F. Are You Ready For More?
G. Stacking Cups

How will you determine which stations individual students participate in? How can you organize the stations in a way that empowers students' development of mathematical identities, provides essential support, and limits the appearance of "smart" or "not-smart" assignments?

Agenda, Materials, and Preparation

- **Station A** (*Required, 20 minutes*)
  - Unit 3 Check Your Readiness (print 1 copy per student)
- **Station B** (*20 minutes*)
  - Formative student data collected from Unit 2 Check Your Readiness and Unit 2 Lessons 1–8
- **Station C** (*20 minutes*)
- **Station D** (*20 minutes*)
  - Weekend Earnings problem #1 (print 1 copy per station)
- **Station E** (*20 minutes*)
- **Station F** (*20 minutes*)
  - Are You Ready For More? tasks in Student Workbook from past lessons (or optional: print 1 blackline master per student)
- **Station G** (*20 minutes*)
  - 10 styrofoam cups per group
  - 1 ruler per group
  - Hint cards (print 1 copy per station)

STATIONS

Station A: Unit 3 Check Your Readiness (*Required, 20 minutes*)

Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Station B: Teacher-led Small-group Instruction (*20 minutes*)

Use student cool-down data, Check Your Readiness Unit 2 data, and informal formative assessment data from Unit 2 (Lessons 1–8) to provide targeted small-group instruction to students who demonstrate the need for further support on topics taught up to this point.

Potential topics:

- Creating equations from context
- Simplifying expressions
- Reasoning about equivalent equations
- Solving multi-step equations

Station C: Let's Go to the Carnival (*20 minutes*)

Addressing: NC.M1.A-CED.1

Students will have the opportunity to apply their knowledge of equations to a real-life scenario. This scenario is ramped: the first question is very accessible, and the degree of difficulty increases as students progress through the problems.

Station C

Tyler and Jada go to a carnival during their summer break from school. They have a total of $50.00 to spend.

Charlotte County Carnival	
Admission	Adults 18+ $10.00 Students $7.50
Game tokens	$1.25 each -or- Pay $20 for unlimited tokens
Concessions	Hotdog $4.25 Waffle Fries $3.75 Ice Cream $2.50 Bottled Water $3.00 Soda or Juice $3.25

1. How much money do Tyler and Jada have to spend after they enter the carnival? Write an equation to show how you arrived at your answer.

2. Do Jada and Tyler have enough money to go to the carnival, play five total games, each eat an ice cream and fries, and share one water bottle?

3. Tyler and Jada run into Lin at the carnival. Lin is trying to decide whether to pay for each game token or buy unlimited tokens. Then she realizes it doesn't matter. How many games must she want to play?

4. For lunch, Tyler and Jada bought two hot dogs, shared an order of fries, and ordered some other items. Their total was $18.25. What else did they buy?

5. Your school is planning a field trip to the carnival! Write an equation that could be used to determine the final price, P, for the number of students, s, and the number of chaperones, c, with each student getting $10 for spending money, and three school buses needed for $110 each.

6. The CMS rules are that one chaperone is needed for every 10 students. If 125 students are going on the field trip, how much will it cost?

DO THE MATH

PLANNING NOTES

Station D: Weekend Earnings (*20 minutes*)

Instructional Routine: Co-Craft Questions (MLR5)	
Addressing: NC.M1.A-CED.1; NC.M1.A-REI.3	

In this station, students write equations in one variable to represent the constraints in a situation. They then reason about the solutions and interpret the solutions in context.

To solve the equation, some students may try different values of h until they find one that gives a true equation. Others may perform the same operations to each side of the equation to isolate h.

- Have students work in pairs or use visibly random grouping to arrange students.
- Provide the blackline master of the prompt and problem 1 for students to read and use the *Co-Craft Questions* routine to create three or four mathematical questions that can be answered using that information. (The remainder of the task will be completed in the Student Workbook.)
  - Prompt: Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay $12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend $7.15 on bus fare.
    - Problem #1: Write three or four mathematical questions that could be solved using this information.
  - Note: if students are unsure how to interpret “take-home earnings,” clarify that it means the amount Jada takes home after paying job-related expenses (in this case, the bus fare).

RESPONSIVE STRATEGIES

Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students after the first 2–3 minutes of work time.

Supports accessibility for: Organization; Attention

Advancing Student Thinking: If students struggle to write equations in the second question, ask them how they might find out Jada's earnings if she works 1 hour, 2 hours, 5 hours, and so on. Then, ask them to generalize the computation process for h hours.

Station D

2. Write an equation that represents Jada’s take-home earnings in dollars, E, if she works at the bookstore for h hours in one day.
3. One day, Jada takes home $90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.
4. Is 4 a solution to the last equation you wrote? What about 7?
   - If so, explain how you know one or both of them are solutions.
   - If not, explain why they are not solutions. Then, find the solution.
5. In this situation, what does the solution to the equation tell us? In other words, what does that mean in terms of Jada’s story?
6. Jada has a second option to earn money: she could help some neighbors with errands and computer work for $11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend. Which option should she choose—sorting books at the bookstore or helping her neighbors? Explain your reasoning.

7. Jada learned that, according to the Census Bureau data from 2018, women in the United States are paid 82 cents for every dollar earned by men.

    a. Write an equation to represent this statistic.

    b. For the day Jada earned $90.45, how much could she have expected to have earned if she were male?

    c. Jada then learned that the statistics showed that the amount women are paid compared to men in the United States is also influenced by race and age. Research the current statistics on wage earnings for women and men in the United States, based on race and age groups. Choose two subgroups and describe the relationship between their expected earnings.

8. As time permits, answer the questions you created in problem #1.

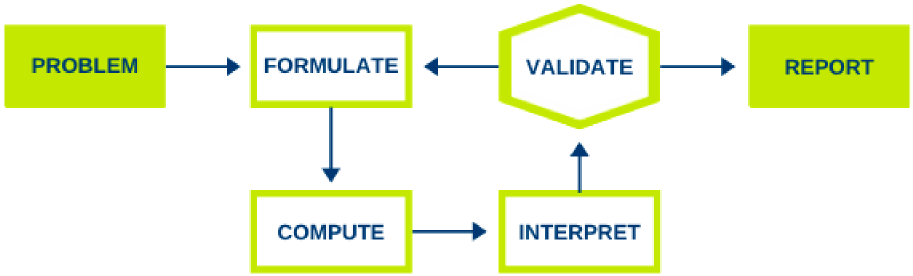
DO THE MATH

PLANNING NOTES

Station E: Micro-Modeling (*20 minutes*)

Instructional Routine: Aspects of Mathematical Modeling

Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling in Math 1 is crucial, as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults regularly encounter in their lives.



These tasks can also be offered as additional practice problems at any point in the unit or used in the teacher-led small-group instruction.

Station E

1. Diego and his family are planning a trip to Carowinds amusement park next Memorial Day weekend. He wants to ride the Afterburn roller coaster, but as of his 9th birthday, October 26th, he is 52.5 inches tall and the ride has a height requirement of 54 inches. His brother, Kiran, was 51.5 inches on his 9th birthday and 54 inches on his 10th birthday. Is it likely that Diego will be able to ride Afterburn when they go next May? Share your reasoning with words, pictures, and/or numbers.

2. For a rock concert, a rectangular field of size 100 meters by 50 meters was reserved for the audience. The concert was completely sold out, and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert? Justify your response by providing a drawing or explanation.[1]

    a. 2,000

    b. 5,000

    c. 20,000

    d. 50,000

    e. 100,000

DO THE MATH

PLANNING NOTES

[1] Adapted from AchievetheCore.org

Station F: Are You Ready For More? (*20 minutes*)

Students who did not complete the “Are You Ready For More?” task statements from Lessons 1, 3, 5, 7, and 8 can do so in Station F. This is a great opportunity for students to expand their thinking. These tasks can also be offered as additional practice problems or used in the teacher-led small-group instruction.

Station F

1. Find a pizza place near you and ask about the diameter and cost of at least two sizes of pizza. Compare the cost per square inch of the sizes.

   (From Unit 2, Lesson 1)

2. Each figure below is created using stars.

   What is the relationship between the figure number and the number of stars in the figure? Represent the relationship in as many ways as possible.

   (From Unit 2, Lesson 3)

figure 1 *figure 2* *figure 3* *figure 4*

3. Here is a puzzle:

   Which expressions could be equal to $p+Q$?

   a. $2p+m$
   b. $4m+N$
   c. $3N$
   d. $9m$

$$\begin{array}{rcl} m+m & = & N \\ N+N & = & p \\ m+p & = & Q \\ p+Q & = & ? \end{array}$$

   (From Unit 2, Lesson 5)

4. Your class is asked to solve this equation.

   $\frac{1}{2}(x+5)=\frac{1}{3}(2x-4)+8$

   Your friend freezes up a bit when they see fractions. Is there an equivalent equation your friend could use that would get the same solution for x but would eliminate the fractions before distributing?

   (From Unit 2, Lesson 7)

5. The movie was so boring, Andre walked out of the theater after seeing only a quarter of it. Fifteen minutes later, Mae walked out after seeing a third of it. How long was the movie?[2]

   (From Unit 2, Lesson 8)

DO THE MATH

PLANNING NOTES

[2] Adapted from AchievetheCore.org

Station G: Stacking Cups[3] (*20 minutes*)

Addressing: NC.M1.A-CED.1; NC.M1.A-REI.3

In this station, students estimate the number of stacked styrofoam cups it will take to reach the top of the teacher's head. To accurately determine this estimate, students must determine the important information in the context and create an equation that models the relationship between number of stacked cups and total height.

Step 1

- Ask students to arrange themselves into groups of three or four or use visibly random grouping.
- For students to successfully complete this station without teacher facilitation, prepare a way to share with students the following information (on the board or digitally):
  - Teacher's height in centimeters (There is a space in their workbook to record this information.)
  - The locations of the following resources:
    - 10 styrofoam cups
    - A ruler
    - Hint cards: 5 cards, each with a question for the group to consider

Station G

In this station, you are tasked with answering the following question:

- How many styrofoam cups would you have to stack to reach the top of your math teacher's head?

Here is some information to get you started:

- My teacher's height is ______ cm.
- A styrofoam cup is approximately ___ cm tall.

You can access the following resources:	You may not:
• 10 styrofoam cups • A ruler • Hints: 5 cards with questions to consider as a small group.	• Hold up the cups next to your teacher to "eyeball" the answer

Step 2

- In the last 5 minutes of the second class sesion, have students stack cups beside the teacher until they reach the top of the teacher's head, counting as they go.
- *Optional*: Award a prize to the group with the closest answer.

DO THE MATH	PLANNING NOTES

[3] Adapted from Dan Meyer

TEACHER REFLECTION

In what ways did the students surprise you over the past two lessons? What were the good surprises? What surprises can you learn from?

What will you want to remember about the challenges from these lessons that will help your planning the next time you do stations?

Lesson 11: Which Variable to Solve For? (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Comprehend that "to solve for a variable" is to rearrange an equation to isolate a variable of interest. • Rearrange multi-variable equations to highlight a particular quantity.	• Given an equation, I can solve for a particular variable (like height, time, or length) when the equation would be more useful in that form. • I know the meaning of the phrase "to solve for a variable."

Lesson Narrative

By now, students are aware that a relationship between two or more quantities can be expressed in multiple ways by writing equivalent equations. In this lesson, they see that, depending on the quantity in which we are interested, one form of equation might be more useful than others. To pin down a quantity of interest may mean manipulating or rearranging a given equation. The rearrangement may involve solving for a variable—isolating it and defining it in terms of the other variables. Students notice that solving for a variable can be an efficient way to solve problems and to avoid cumbersome calculations.

Throughout the lesson, students reason repeatedly and look for regularity as they manipulate equations and solve for a variable (MP8).

What teaching strategies will you be focusing on in this lesson?

Focus and Coherence

Addressing
NC.M1.A-CED.4: Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*10 minutes*)
  - Tools for each student for creating a visual display that can be shared: for example, chart paper and 2 different colored markers per group of 2 students, whiteboard space and 2 different colored markers per group of 2 students, shared online drawing tool, or access to a document camera and 2 different colored pencils per group of 2 students
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L11 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building Towards: NC.M1.A-CED.4

The purpose of this bridge is for students to begin thinking about how they can manipulate a formula to find the value of any variable. This will be useful later in this lesson when students solve formulas and equations to isolate specific variables.

Student Task Statement

Use the formula for the area of a rectangle, $A = lw$, to answer the following questions.

As you work, look for patterns or a set of steps that you could use to quickly figure out one measurement, given the others.

1. A rectangle has a length of 4 units and a width of 9 units. Find its area.
2. A rectangle has an area of 50 square units and a width of 10 units. Find its length.
3. A rectangle has an area of 56 square units and a width of 7 units. Find its length.
4. A rectangle has an area of 64 square units and a width of 4 units. Find its length.
5. How would you tell someone to find the measurement of a rectangle's length when given its area and width?

DO THE MATH	PLANNING NOTES

Warm-up: Which Equations? (*5 minutes*)

Building Towards: NC.M1.A-CED.4

In this warm-up, students look for a relationship between two quantities by interpreting a verbal description and analyzing pairs of values in a table. They then use the observed relationship to find unknown values of one quantity given the other, and to think about possible equations that could represent the relationship more generally (MP8).

The work here reinforces the idea that the relationship between two quantities can be expressed in more than one way, and that some forms might be more helpful than others, depending on what we want to know. In this context, for instance, if we know the area of the parallelogram and want to know its base length, the equation $b=\frac{A}{3}$ is more helpful than $A=3b$

Step 1

- Offer students the choice to work independently or with a partner to solve the warm-up.

Advancing Student Thinking: Some students may think that the height must be known before they could find the missing area or base. Encourage them to look for a pattern in the table and to reason from there.

Student Task Statement

1. The table shows the relationship between the base length, b, and the area, A, of some parallelograms. All the parallelograms have the same height. Base length is measured in inches, and area is measured in square inches. Complete the table.

b (inches)	A (square inches)
1	3
2	6
3	9
4.5	
$\frac{11}{2}$	
	36
	46.5

2. Decide whether each equation could represent the relationship between b and A. Explain your reasoning.

    a. $b=3A$

    b. $b=\frac{A}{3}$

    c. $A=\frac{b}{3}$

    d. $A=3b$

Step 2

- Facilitate a whole-class discussion by inviting students to share their responses and explanations. Discuss with students:
  - "Are the two equations that were chosen equivalent? How do you know?" (Yes. There is an acceptable move that takes one to the other. If we divide each side of $A = 3b$ by 3, we have $\frac{A}{3} = b$, which can also be written as $b = \frac{A}{3}$. If we multiply each side of $b = \frac{A}{3}$ by 3, we have $3b = A$ or $A = 3b$.)
  - "If we know the base, which equation would make it easier to find the area? Why?" ($A = 3b$. The variable for area is already isolated. All we have to do is multiply the base by 3 to find the area.)
  - "If we know the area, which equation would make it easier to find the base? Why?" ($b = \frac{A}{3}$. The variable for the base is already isolated. We can just divide the area by 3 to find the base.)

DO THE MATH	PLANNING NOTES

Activity 1: Rewriting Formulas (*10 minutes*)

Instructional Routine: Collect and Display (MLR2)
Addressing: NC.M1.A-CED.4

The goal of this activity is for students to practice rearranging equations to highlight a quantity of interest, using the same reasoning they use for solving equations. Students should recognize most of the formulas given but may not know exactly what, say, $E = mc^2$ represents. Reassure students that they can skip the interpretations of formulas that they don't know, and if time allows, they can talk about these formulas during the debrief.

Step 1

- Offer students the choice to work independently or with a partner to complete this task.
- Ask students to be prepared to share their reasoning around their rewritten equations, paying attention to acceptable moves.
- As students work, use the *Collect and Display* routine. Circulate and listen to pairs talk and collect any phrases or words that communicate about acceptable moves and about equivalence.

Student Task Statement

Equation or formula	What does this formula represent?	Solve for: (Assume no variable is equal to 0)
$d = rt$		r
$d = rt$		t
$y = mx + b$		x
$A = \frac{bh}{2}$		b
$P = 2l + 2w$		w
$E = mc^2$		m
$F = \frac{9}{5}C + 32$		C
$(y - y_1) = m(x - x_1)$		m

Step 2

- Display any language collected from students as they worked and refer to this language during facilitation of a whole-class discussion.
  - Select students to present their strategies, rewritten equations, and their reasoning for why the two equations in a given row are equivalent to each other.
  - Emphasize that isolating the variable of interest can be an efficient way for solving problems. Once this variable of interest is pinned down first, students can see what expression is equal to it and then simply evaluate that expression, bypassing some tedious steps.
  - Highlight that isolating a variable is called "solving for a variable."

DO THE MATH	PLANNING NOTES

Activity 2: Filling and Emptying Tanks (*10 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Addressing: NC.M1.A-CED.4; NC.M1.A-REI.3	

In this activity, students solve problems involving two quantities in non-proportional linear relationships. As before, they are prompted to reason repeatedly about the value of one quantity given the other, and to generalize the process by writing an expression (MP8). They then connect the work here to the idea of writing an equation and isolating a variable of interest.

As students work, monitor for the different ways students reason about the unknown number of minutes in the second question (for tank A) and the last question (for tank B). Some students may find the missing value by reasoning about the quantities informally. Others may write an equation and reason about the equations, or solve the equation for a variable.

For example, to find the number of minutes that have passed when tank B has 18 liters left, students may:

- Reason that the tank has lost 62 liters because $80-62=18$ or $80-18=62$. If each minute, it loses 2.5 liters, then it would take $\frac{62}{2.5}$ or 24.8 minutes for it to lose 62 liters.
- Write $80-2.5t=18$ and think: "I'm looking for a number that, when multiplied by 2.5 and subtracted from 80, gives 18," or "I'm looking for a number that, when multiplied by 2.5, is equal to $80-18$."
- Write $80-2.5t=18$ and solve for t this way: Or this way:

$$\begin{aligned} 80-2.5t &= 18 \\ 80-18-2.5t &= 0 \\ 80-18 &= 2.5t \\ \frac{80-18}{2.5} &= t \\ \frac{62}{2.5} &= t \\ 24.8 &= t \end{aligned}$$

$$\begin{aligned} 80-2.5t &= 18 \\ -2.5t &= 18-80 \\ t &= \frac{18-80}{-2.5} \\ t &= \frac{-62}{-2.5} \\ t &= \frac{62}{2.5} \\ t &= 24.8 \end{aligned}$$

Regardless of the approach students take, the important idea to spotlight (for tank B) is that finding the time at which the water in the tank reaches a certain volume can be done by subtracting that volume (v) from the original volume (80 liters) and then dividing it by 2.5. That process can be summarized by the expression: $\frac{80-v}{2.5}$.

If we start out with the equation $80-2.5t=v$ and perform allowable moves to isolate t, we will end up with $t=\frac{80-v}{2.5}$.

RESPONSIVE STRATEGIES

Leverage choice around perceived challenge. Invite students to choose between working on questions about tank A or the questions about tank B. Chunking this task into more manageable parts may also support students who benefit from additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Step 1

- Ask students to arrange themselves in groups of two if they aren't already paired up or use visibly random grouping.
- Ask one half of the class to answer the first two questions about tank A and the other half to answer the last two questions about tank B.

Advancing Student Thinking: For students who struggle to write expressions for p liters and v liters, encourage them to revisit their previous three calculations. Some students may need to write out their work more carefully before noticing that they could perform the same steps using p or w in place of a number.

Student Task Statement

Tank A initially contained 124 liters of water. It is then filled with more water, at a constant rate of 9 liters per minute.

1. How many liters of water are in tank A after the following amounts of time have passed?
   a. 4 minutes
   b. 80 seconds
   c. m minutes
2. How many minutes have passed, m, when tank A contains the following amounts of water?
   a. 151 liters
   b. 191.5 liters
   c. 270.25 liters
   d. p liters

Tank B, which initially contained 80 liters of water, is being drained at a rate of 2.5 liters per minute.

3. How many liters of water remain in the tank after the following amounts of time?
   a. 30 seconds
   b. 7 minutes
   c. t minutes
4. For how many minutes, t, has the water been draining when tank B contains the following amounts of water?
   a. 75 liters
   b. 32.5 liters
   c. 18 liters
   d. v liters

Step 2

- In preparation for a whole-class discussion, invite pairs of students to create a visual display of their work about either tank A or tank B. Ask them to display their work so that other students will be able to interpret the connections between their calculations and the expressions they created, and to add notes or other details to their displays to help communicate their thinking.

Step 3

- Use the *Compare and Connect* routine to structure a whole-class discussion about the expressions for the last part of each question. Begin the whole-class discussion by selecting and arranging a few student displays for all to see. Give students 1–2 minutes of quiet think time to interpret the displays before inviting the authors to share their expressions.
- Invite students to share their expressions for the last part of each question. Then, facilitate a discussion on how the different strategies students used to answer the other parts of each question could be summed up by their expression.

- Discuss questions such as:
  - "How do you know that the expression gives us the liters of water in tank A after m minutes?" (When finding the liters of water in the tank after 4 minutes and 80 seconds, we multiplied the minutes by 9 and then added it to 124. So for m minutes, we'd multiply m by 9 and add 124.)
  - "How do you know that the expression gives us the minutes at which the tank reaches p liters?" (When finding the minutes at which the tank reaches 151 liters, 191.5 liters, and 270.25 liters, we subtracted 124 from each number then divided the difference by 9. So we can do the same for p liters.)
- If no students mentioned that they found the expression for the last part of the second and last questions by writing an equation and solving it for the variable of interest, demonstrate it for all to see.

  For example, in tank A, we know the relationship between the liters of water in the tank, p, after m minutes is $124 + 9m = p$. To find the minutes at which the tank reaches p liters, we can isolate m:

$$\begin{aligned} 124 + 9m &= p \\ 9m &= p - 124 \\ m &= \frac{p-124}{9} \end{aligned}$$

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students learned the meaning of “solving for” a variable. They saw that isolating a variable can be a useful way to avoid repeated calculations. While facilitating the discussion, encourage students both to explain how to solve for a variable and to talk about why they might want to do so.

To help students consolidate and reflect on their work in the lesson, present the following scenario.

Suppose a classmate who is absent today wanted to know:

- What does it mean to "solve for a variable"?
- Why should we solve for a variable?
- How do we solve for a variable?

Ask students: "How would you respond to these questions and help your classmate catch up with what was missed?"

Consider arranging students in groups of two and asking partners to do a role play, taking turns being the absent classmate and the explainer or have students respond in their workbooks. Display the equations from the lesson for all to see. Encourage students to use one or two of them to demonstrate how to solve for a variable.

$A = 3b$

$d = rt$

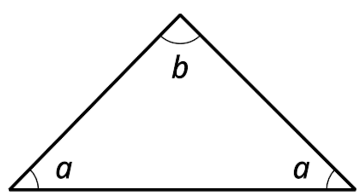
$y = mx + b$

$124 + 9m = p$

PLANNING NOTES

Student Lesson Summary and Glossary

A relationship between quantities can be described in more than one way. Some ways are more helpful than others, depending on what we want to find out. Let’s look at the angles of an isosceles triangle, for example.



The two angles near the horizontal side have equal measurement in degrees, a.

The sum of angles in a triangle is $180°$, so the relationship between the angles can be expressed as:

$$a + a + b = 180$$

Suppose we want to find a when b is $20°$.

Let's substitute 20 for b and solve the equation.

$$\begin{aligned} a + a + b &= 180 \\ 2a + 20 &= 180 \\ 2a &= 180 - 20 \\ 2a &= 160 \\ a &= 80 \end{aligned}$$

What is the value of a if b is $45°$?

$$\begin{aligned} a+a+b &= 180 \\ 2a+45 &= 180 \\ 2a &= 180-45 \\ 2a &= 135 \\ a &= 67.5 \end{aligned}$$

Now suppose the bottom two angles are $34°$ each. How many degrees is the top angle?

Let's substitute 34 for a and solve the equation.

$$\begin{aligned} a+a+b &= 180 \\ 34+34+b &= 180 \\ 68+b &= 180 \\ b &= 112 \end{aligned}$$

What is the value of b if a is $72.5°$?

$$\begin{aligned} a+a+b &= 180 \\ 72.5+72.5+b &= 180 \\ 145+b &= 180 \\ b &= 35 \end{aligned}$$

Notice that when b is given, we did the same calculation repeatedly to find a: we substituted b into the first equation, subtracted b from 180, and then divided the result by 2.

Instead of taking these steps over and over whenever we know b and want to find a, we can rearrange the equation to isolate a:

$$\begin{aligned} a+a+b &= 180 \\ 2a+b &= 180 \\ 2a &= 180-b \\ a &= \frac{180-b}{2} \end{aligned}$$

This equation is equivalent to the first one. To find a, we can now simply substitute any value of b into this equation and evaluate the expression on the right side.

Likewise, we can write an equivalent equation to make it easier to find b when we know a:

$$\begin{aligned} a+a+b &= 180 \\ 2a+b &= 180 \\ b &= 180-2a \end{aligned}$$

Rearranging an equation to isolate one variable is called “solving for a variable.” In this example, we have solved for a and for b. All three equations are equivalent. Depending on what information we have and what we are interested in, we can choose a particular equation to use.

Cool-down: A Rectangular Relationship (*5 minutes*)

Addressing: NC.M1.A-CED.4; NC.M1.A-REI.3

Cool-down Guidance: More Chances
Lesson 12 offers more chances to practice this.

Cool-down

The perimeter of a rectangle is 48 centimeters. The relationship between the length, the width, and the perimeter of the rectangle can be described with the equation $2 \cdot length + 2 \cdot width = 48$.

Find the length, in centimeters, if the width is:

1. 10 centimeters
2. 3.6 centimeters
3. w centimeters

Student Reflection: What is at least one thing you did differently within the past week to improve in math class?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Revisit the norms you established as a class about doing mathematics. Which norms are working and which might need revision? Are there any norms you or your students might want to add?

Practice Problems

1. Priya is buying raisins and almonds to make trail mix. Almonds cost $5.20 per pound and raisins cost $2.75 per pound. Priya spent $11.70 buying almonds and raisins. The relationship between pounds of almonds a, pounds of raisins, r, and the total cost is represented by the equation $5.20a + 2.75r = 11.70$.

   How many pounds of raisins did Priya buy if she bought the following amounts of almonds:

   a. 2 pounds of almonds

   b. 1.06 pounds of almonds

   c. 0.64 pounds of almonds

   d. a pounds of almonds

2. Here is a linear equation in two variables: $2x + 4y - 31 = 123$. Solve the equation, first for x and then for y.

3. A chef bought $\$17.01$ worth of ribs and chicken. Ribs cost $\$1.89$ per pound, and chicken costs $\$0.90$ per pound. The equation $0.9c + 1.89r = 17.01$ represents the relationship between the quantities in this situation.

   Show that each of the following equations is equivalent to $0.9c + 1.89r = 17.01$. Then, explain when it might be helpful to write the equation in these forms.

   a. $c = 18.9 - 2.1r$

   b. $r = -\frac{10}{21}c + 9$

4. A car traveled 180 miles at a constant rate.

   a. Complete the table to show the rate at which the car was traveling if it completed the same distance in each number of hours.

Travel time (hours)	Rate of travel (miles per hour)
5	
4.5	
3	
2.25	

   b. Write an equation that would make it easy to find the rate at which the car was traveling in miles per hour, r, if it traveled for t hours.

5. Select **all** the equations that are equivalent to the equation $3x - 4 = 5$.

   a. $3x = 9$
   b. $3x - 4 + 4 = 5 + 4$
   c. $x - 4 = 2$
   d. $x = 9$
   e. $-4 = 5 - 3x$

   (From Unit 2, Lesson 5)

6. Elena says that equations a and b are not equivalent.

   a. $13 - 5x = 48$

   b. $5x = 35$

   Write a convincing explanation as to why this is true.

   (From Unit 2, Lesson 6)

7. Han is solving an equation. He took steps that are acceptable but ended up with equations that are clearly not true.

$$\begin{aligned} 5x+6 &= 7x+5-2x && \textit{original equation} \\ 5x+6 &= 7x-2x+5 && \textit{apply the commutative property} \\ 5x+6 &= 5x+5 && \textit{combine like terms} \\ 6 &= 5 && \textit{subtract } 5x \textit{ from each side} \end{aligned}$$

   What can Han conclude as a result of these acceptable steps?

   a. There's no value of x that can make the equation $5x+6=7x+5-2x$ true.

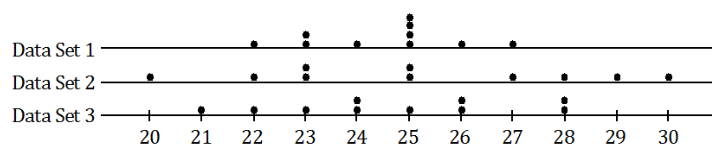
   b. Any value of x can make the equation $5x+6=7x+5-2x$ true.

   c. $x=6$ is a solution to the equation $5x+6=7x+5-2x$.

   d. $x=5$ is a solution to the equation $5x+6=7x+5-2x$.

   (From Unit 2, Lesson 6)

8. Three data sets are shown in the dot plots below.[1]



   a. Which data set has the smallest standard deviation of the three? Justify your answer.

   b. Which data set has the largest standard deviation of the three/? Justify your answer.

   (From Unit 1)

9. Suppose that a teacher plans to give four students a quiz.[2] The minimum possible score on the quiz is 0, and the maximum possible score is 10.

   a. What is the smallest possible standard deviation of the students' scores? Give an example of a possible set of four student scores that would have this standard deviation.

   b. What is the set of four student scores that would make the standard deviation as large as it could possibly be? Use a calculator or Desmos to find this largest possible standard deviation.

   (From Unit 1)

10. The formula for the area of a triangle is $A = \frac{1}{2}bh$.

   a. A triangle has a base of 5 units and a height of 4 units. Find its area.

   b. A triangle has an area of 12 square units and a height of 8 units. Find its base.

   c. How would you tell someone to find the length of a triangle's base when given its area and height?

   (Building towards NC.M1.A-CED.4)

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[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

Lesson 12: Which Variable to Solve For? (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Practice writing equations in two or more variables and solving for a particular variable. • Solve for a variable by performing acceptable operations, including when the values of other quantities in a multi-variable equation are not known.	• I can write an equation to describe a situation that involves multiple quantities whose values are not known and then solve the equation for a particular variable. • I know how solving for a variable can be used to quickly calculate the values of that variable.

Lesson Narrative

This lesson builds off of the previous lesson where students learned that it is possible, and sometimes preferable, to first solve for a specific variable before substituting known values or performing calculations.

In this lesson, students practice solving for the variable and obtaining an expression that defines it in terms of the others. They see that doing so allows them to solve problems more efficiently.

Computer applications such as Desmos can help illustrate the benefits of isolating a variable. Students see that once a variable of interest is isolated and expressed in terms of all the other variables, a computer program can use that expression (even if it seems complicated) and speedily calculate its value when the other variables take on different values. The interactive model is a powerful way to test different assumptions in a situation (MP4).

How is the approach of this lesson similar and different from other ways you have taught these concepts or procedures?

Focus and Coherence

Addressing
NC.M1.A-CED.4: Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*25 minutes*)
  - Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device.
  - Desmos User Guide pages 3 and 4 (print 1 copy per student)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L12 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building Towards: NC.M1.A-CED.4

The purpose of this bridge is to support students in understanding why a particular equation represents a situation that has two unknown quantities. Students build on their ability to find the value of one missing quantity when given the value of the other. This helps to focus students' attention on equivalent forms of equations in two variables: $y = 36 - 19$ prompts students to generalize to $y = 36 - x$. This will be useful when students learn to solve equations for a variable when the values of other quantities in a multi-variable equation are not known.

Student Task Statement

Sparkling water and grape juice are mixed together to make 36 ounces of fizzy juice.

1. How much sparkling water is used if the mixture contains 19 ounces of grape juice?
2. Han wrote the equation $x + y = 36$, with x representing the amount of grape juice used, in ounces, and y representing the amount of sparkling water used, in ounces. Explain why Han's equation matches the story.

DO THE MATH	PLANNING NOTES

Warm-up: Faces, Vertices, and Edges (*5 minutes*)

Addressing: NC.M1.A-CED.4

In this warm-up, students are given an equation in three variables and are prompted to rearrange it to pin down a particular variable. In each question, only the value of one variable is given, so students need to manipulate the equation even when some quantities are unknown. The work here prepares students to rearrange other variable equations later in the lesson.

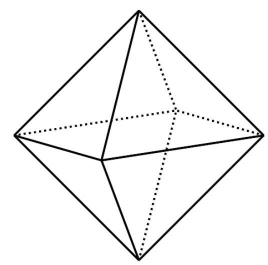
Step 1

- Offer students the opportunity to work independently or with a partner to complete the warm-up.

> **Monitoring Tip:** As students work, look for those who substitute the given value before rearranging and those who first isolate the variable of interest before substituting. Invite them to share their approaches during class discussion.

Student Task Statement

The equation $V+F-2=E$ relates the number of vertices (V), faces (F), and edges (E) in a Platonic solid.



1. Write an equation that makes it easier to find the number of vertices in each of the Platonic solids described:
   - an octahedron (shown here), which has 8 faces
   - an icosahedron, which has 30 edges
2. A Buckminsterfullerene (also called a "Buckyball") is a polyhedron with 60 vertices. It is not a Platonic solid, but the numbers of faces, edges, and vertices are related the same way as those in a Platonic solid.

   Write an equation that makes it easier to find the number of faces a Buckyball has if we know how many edges it has.

Step 2

- Select previously identified students to share their responses and strategies. Record and display for all to see the steps they take to rearrange the equations. Emphasize how each step constitutes an acceptable move and how it keeps the equation true.
- Make sure students see that known values can be substituted into the given equation before rearranging it, or the equation can be rearranged first before substituting known values. In the examples here, it doesn't matter which way it is done. Ultimately, they were solving for V in the first question and for F in the second question.
- Explain that there will be times when one strategy might be more helpful than the other, as students will see in subsequent activities.

DO THE MATH	PLANNING NOTES

Activity 1: Cargo Shipping (*25 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Three Reads (MLR6); Co-Craft Questions (MLR5)
Addressing: NC.M1.A-CED.3; NC.M1.A-CED.4; NC.M1.A-REI.3

This activity encourages students to write an equation in two variables to represent a constraint and then solve for each of the variables. One motivation for rearranging the equation is to find an expression that, when entered into a calculator or computer, can then be used to quickly find the value of one quantity given the value of the other.

Students engage in *Aspects of Mathematical Modeling* (MP4) as they use spreadsheet technology to create mathematical models, test them, and solve problems.

Step 1

- Use the *Three Reads* routine together with *Co-Craft Questions* with the first two lines of the task, before students look at the questions.

> ***Why This Routine?*** *Three Reads* (MLR6) gives students a chance to use everyday language to help each other make sense of the context -- and the language -- of a word problem before jumping down a solution path. Use this routine to ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the general structure of quantitative situations and on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with peers through mathematical conversation.

  - First Read: Read the situation aloud to students.
    - Ask: "What is this situation about? Focus on what is going on, not on the numbers."
    - Spend less than 1 minute scribing students' understanding of the situation. Clarify the meaning of any unfamiliar words as needed (e.g., cargo ship, load).
  - Second Read: Display the task and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Again, spend less than a minute scribing student responses. Encourage students to identify quantities that are named in the problem explicitly and any quantities that may be implicit.
  - Third Read: Invite students to read the situation again to themselves, or ask another student volunteer to read it aloud.
    - Ask: "What mathematical questions could we ask?"
    - Spend 1–2 minutes using the *Co-Craft Questions* routine scribing student ideas as they brainstorm possible questions.

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students a few minutes of quiet time to think about the first question and then time to discuss their responses with their partner.
- Ask students to pause for a class discussion, and ensure everyone is using a correct equation before proceeding.

> **RESPONSIVE STRATEGIES**
>
> Provide students with a physical copy of written directions for using the two Desmos options and read them aloud. Include step-by-step directions for how to enter equations, find the value of one variable by entering the value of another, and test equations.
>
> Supports accessibility for: Language; Memory

Step 3

- Once students have responses for the second question, invite them to share how they found the number of cars that can be shipped if the cargo already has some number of trucks. Next, ask for the expressions they wrote to find the number of cars that can fit if there are t trucks. Record the expressions for all to see.
- Tell students that they can test the expressions by using a computer. Demonstrate how to use technology to calculate the number of cars given the number of trucks using the table function in Desmos: https://www.desmos.com/calculator/pbkc6levg0, or using the slider function: https://www.desmos.com/calculator/2porjwvdm3.
- Remind students that their job in the last question is to find the number of trucks when the number of cars is known. Encourage them to test their equations using available technology.

Provide students with a physical copy of the Desmos User Guide or access it digitally at http://bit.ly/DesmosUserGuide for using the two Desmos options and read them aloud. Review directions on pages 3 and 4 of the user guide for how to enter equations, find the value of one variable by entering the value of another, and test equations.

Student Task Statement

An automobile manufacturer is preparing a shipment of cars and trucks on a cargo ship that can carry 21,600 tons.

The cars weigh 3.6 tons each, and the trucks weigh 7.5 tons each.

1. Write an equation that represents the weight limit of a shipment. Let c be the number of cars and t be the number of trucks.
2. For one shipment, trucks are loaded first, and cars are loaded afterwards. (Even though trucks are bulkier than cars, a shipment can consist of all trucks as long as it is within the weight limit.)

   Find the number of cars that can be shipped if the cargo already has:

   a. 480 trucks

   b. 1,500 trucks

   c. 2,736 trucks

   d. t trucks
3. For a different shipment, cars are loaded first, and then trucks are loaded afterwards.

   a. Write an equation you could use to find the number of trucks that can be shipped if the number of cars is known.

   b. Use your equation to find the number of trucks that can be shipped if the cargo already has 1,000 cars. What if the cargo already has 4,250 cars?

Are You Ready For More?

For yet another shipment, the manufacturer is also shipping motorcycles, which weigh 0.3 ton each.

1. Write an equation that you could enter into a calculator or a spreadsheet tool to find the number of motorcycles that can be shipped, m, if the number of cars and trucks are known.
2. Use your equation to find the number of motorcycles that can be shipped if the cargo already contains 1,200 trucks and 3,000 cars.

Step 3

- When finding the number of cars, c, given t trucks, students may have arrived at the expression $\frac{21{,}600-7.5t}{3.6}$ by generalizing the calculation they performed when the number of trucks was a numerical value. Likewise, in the last question, they may have arrived at $t=\frac{21{,}600-3.6c}{7.5}$ by using some numerical values for c and generalizing the process.
- While this strategy is expected and perfectly reasonable, make sure students also see that we can arrive at the same expression for c and for t by rearranging the equation $3.6c+7.5t=21,600$.
- Highlight that we can solve for c when we know the number of trucks and want to compute the number of cars, and solve for t when we know the number of cars and want to find the number of trucks.

$$\begin{aligned} 3.6c+7.5t &= 21,600 \\ 3.6c &= 21,600-7.5t \\ c &= \frac{21{,}600-7.5t}{3.6} \end{aligned}$$

$$\begin{aligned} 3.6c+7.5t &= 21,600 \\ 7.5t &= 21,600-3.6c \\ t &= \frac{21{,}600-3.6c}{7.5} \end{aligned}$$

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this debrief is for students to come up with reasons why they might want to solve for one variable rather than another. Students also get the opportunity to practice using algebra to isolate a variable.

Choose whether students should first have an opportunity to reflect on the following question in their workbooks or talk through them with a partner.

Describe and display the following situation to students.

Suppose you are organizing a party and have a budget of b dollars for the appetizers. You plan to order v vegetarian spring rolls at $\$0.75$ each and s shrimp rolls at $\$0.95$ each. The equation $0.75v+0.95s=b$ represents this constraint.

Ask one half of the class to solve for v and the other half to solve for s.

PLANNING NOTES

Then, ask students: "When might it be most handy to use each of these equations in your party planning?"

- $0.75v + 0.95s = b$ (when we want to find out how much different combinations of rolls would cost)
- $v = \frac{b-0.95s}{0.75}$ (when we know the budget and want to find out how many vegetarian rolls we could get if we order different numbers of shrimp rolls)
- $s = \frac{b-0.75v}{0.95}$ (when we know the budget and want to find out how many shrimp rolls we could get if we order different numbers of vegetarian rolls)

Student Lesson Summary and Glossary

Solving for a variable is an efficient way to find out the values that meet the constraints in a situation. Here is an example:

An elevator has a capacity of $3,000$ pounds and is being loaded with boxes of two sizes—small and large. A small box weighs 60 pounds, and a large box weighs 150 pounds.

Let x be the number of small boxes and y the number of large boxes. To represent the combination of small and large boxes that fill the elevator to capacity, we can write:

$$60x + 150y = 3,000$$

If there are 10 large boxes already, how many small boxes can we load onto the elevator so that it fills it to capacity? What if there are 16 large boxes?

In each case, we can substitute 10 or 16 for y and perform acceptable moves to solve the equation. Or, we can first solve for x:

$$\begin{aligned} 60x + 150y &= 3,000 && \text{original equation} \\ 60x &= 3,000 - 150y && \text{subtract 150y from each side} \\ x &= \tfrac{3,000-150y}{60} && \text{divide each side by 60} \end{aligned}$$

This equation allows us to easily find the number of small boxes that can be loaded, x, by substituting any number of large boxes for y.

Now suppose we first load the elevator with small boxes, say, 30 or 42, and want to know how many large boxes can be added for the elevator to reach its capacity.

We can substitute 30 or 42 for x in the original equation and solve it. Or, we can first solve for y:

$$\begin{aligned} 60x + 150y &= 3,000 && \text{original equation} \\ 150y &= 3,000 - 60x && \text{subtract 60x from each side} \\ y &= \tfrac{3,000-60x}{150} && \text{divide each side by 150} \end{aligned}$$

Now, for any value of x, we can quickly find y by evaluating the expression on the right side of the equal sign.

Solving for a variable—before substituting any known values—can make it easier to test different values of one variable and see how they affect the other variable. It can save us the trouble of doing the same calculation over and over.

Cool-down: Carnival Tickets (*5 minutes*)

Addressing: NC.M1.A-CED.4; NC.M1.A-REI.3

Cool-down Guidance: More Chances.
How to solve for a variable is a point to emphasize and will come up again moving into inequalities and as students graph equations in Unit 3.

Cool-down

A school is holding a carnival and hopes to raise \$500. Child tickets cost \$3, and adult tickets cost \$5. If the school sells x child tickets and y adult tickets, then the equation $3x + 5y = 500$ expresses the fact that the school raised exactly \$500 from ticket sales.

1. Solve the equation for x.
2. Explain when it might be helpful to rewrite the equation this way.

Student Reflection: Today my participation was (circle one): *high* *medium* *low* because __________________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What question do you wish you had asked today? When and why should you have asked it?

Practice Problems

1. A car has a 16-gallon fuel tank. When driven on a highway, it has a gas mileage of 30 miles per gallon. The gas mileage (also called "fuel efficiency") tells us the number of miles the car can travel for a particular amount of fuel (one gallon of gasoline, in this case). After filling the gas tank, the driver got on a highway and drove for a while.

    a. How many miles has the car traveled if it has the following amounts of gas left in the tank?

    - 15 gallons
    - 10 gallons
    - 2.5 gallons

    b. Write an equation that represents the relationship between the distance the car has traveled in miles, d, and the amount of gas left in the tank in gallons, x.

    c. How many gallons are left in the tank when the car has traveled the following distances on the highway?

    - 90 miles
    - 246 miles

    d. Write an equation that makes it easier to find the the amount of gas left in the tank, x, if we know the car has traveled d miles.

2. The area A of a rectangle is represented by the formula $A = lw$ where l is the length and w is the width. The length of the rectangle is 5.

    Write an equation that makes it easy to find the width of the rectangle if we know the area and the length.

3. Noah is helping to collect the entry fees at his school's sports game. Student entry costs $\$2.75$ each, and adult entry costs $\$5.25$ each. At the end of the game, Diego collected $\$281.25$.

    Select **all** equations that could represent the relationship between the number of students, s, the number of adults, a, and the dollar amount received at the game.

    a. $281.25 - 5.25a = 2.75s$

    b. $a = \frac{281.25 - 2.75s}{5.25}$

    c. $281.25 - 5.25s = a$

    d. $281.25 + 2.75a = s$

    e. $281.25 + 5.25s = a$

4. $V = \pi r^2 h$ is an equation to calculate the volume of a cylinder, V, where r represents the radius of the cylinder and h represents its height.

    Which equation allows us to easily find the height of the cylinder because it is solved for h?

    a. $r^2 h = \frac{V}{\pi}$
    b. $h = V - \pi r^2$
    c. $h = \frac{V}{\pi r^2}$
    d. $\pi h = \frac{V}{r^2}$

5. The Department of Streets of a city has a budget of $1,962,800 for resurfacing roads and hiring additional workers this year.

   The cost of resurfacing a mile of two-lane road is estimated at $84,000. The average starting salary of a worker in the department is $36,000 a year.

   a. Write an equation that represents the relationship between the miles of two-lane roads the department could resurface, m, and the number of new workers it could hire, p, if it spends the entire budget.

   b. Take the equation you wrote in the first question and:

      - Solve for p. Explain what the solution represents in this situation.
      - Solve for m. Explain what the solution represents in this situation.

   c. The city is planning to hire six new workers and to use its entire budget.

      - Which equation should be used to find out how many miles of two-lane roads it could resurface? Explain your reasoning.
      - Find the number of miles of two-lane roads the city could resurface if it hires six new workers.

6. A car is going 65 miles per hour down the highway.

   a. How far does it travel in 1.5 hours?

   b. How long does it take the car to travel 130 miles?

   c. Mai wrote the equation $y = 65x$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Mai's equation matches the story.

   (From Unit 2, Lesson 3)

7. The table shows the volume of water in cubic meters, V, in a tank after water has been pumped out for a certain number of minutes. Which equation could represent the volume of water in cubic meters after t minutes of water being pumped out?

Time after pumping begins	Volume of water (cubic meters)
0	30
5	27.5
10	20
15	7.5

   a. $V = 30 - 2.5t$
   b. $V = 30 - 0.5t$
   c. $V = 30 - 0.5t^2$
   d. $V = 30 - 0.1t^2$

   (From Unit 2, Lesson 4)

8. Which equation has the same solution as $10x - x + 5 = 41$?

   a. $10x + 5 = 41$
   b. $10x - 5 + x = 41$
   c. $9x = 46$
   d. $9x + 5 = 41$

   (From Unit 2, Lesson 5)

9. Noah is solving an equation and one of his moves is unacceptable. Here are the moves he made.

$$\begin{array}{rl} 2(x+6) = 8+6x & \text{original equation} \\ 2x+12-4 = 8+6x & \text{apply the distributive property} \\ 2x+8 = 8+6x & \text{combine like terms} \\ 2x = 6x & \text{subtract 8 from both sides} \\ 2 = 6 & \text{divide each side by x} \end{array}$$

Which answer **best** explains why the "divide each side by x step" is unacceptable?

a. When you divide both sides of $2x = 6x$ by x, you get $2x^2 = 6x^2$.
b. When you divide both sides of $2x = 6x$ by x, it could lead us to think that there is no solution while in fact the solution is $x = 0$.
c. When you divide both sides of $2x = 6x$ by x, you get $2 = 6x$.
d. When you divide both sides of $2x = 6x$ by x, it could lead us to think that there is no solution while in fact the solution is $x = 3$.

(From Unit 2, Lesson 6)

10. This data set represents the number of hours 10 students slept on Sunday night.

6	6	7	7	7	8	8	8	8	9

Are there any outliers? Explain your reasoning.

(From Unit 1)

Lessons 13 & 14: Mathematical Modeling[1]

PREPARATION

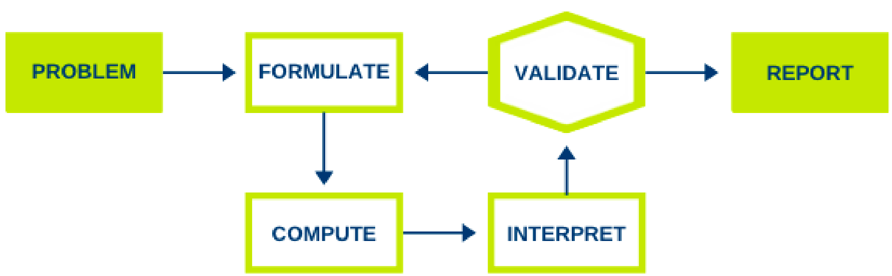
Lesson Goals	Learning Targets
• Understand the meaning of a modeling prompt. • Understand expectations of a student response to a modeling prompt.	• I can describe what a modeling prompt is. • I can explain some elements of a good response to a modeling prompt.

Lesson Narrative

Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling is crucial, as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults regularly encounter in their lives.

In addition to micro-modeling opportunities on the Checkpoint Lesson days, there will be four scheduled modeling blocks (two lessons each) over the course of Math 1. Lessons 13 & 14 are the first of those four. In each of the blocks, there will be teacher choice around which modeling prompt(s) to offer and which version of the prompt(s) students will receive. Many prompts offer different versions that provide varying levels of complexity. However, it is highly recommended to begin with Modeling Prompt #1, which provides students a scaffolded way to learn about modeling by evaluating a sample response to a prompt. If time permits, Modeling Prompt #2 is also offered in which students either name a question that requires them to gather and analyze data to answer or are given a data set to work with.

Students most likely will have had little opportunity to do much modeling prior to Math 1, so taking time to support the ideas around the modeling cycle is important. Students may be surprised to be presented with a problem where they need to make assumptions or that there are multiple "correct" responses.



Things the Modeler Does When Modeling with Mathematics (NGA 2010)

1. *Pose a problem* that can be explored with quantitative methods. Identify variables in the situation and select those that represent essential features.
2. *Formulate a model*: Create and select geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between variables.
3. *Compute*: Analyze these relationships and perform computations to draw conclusions.
4. *Interpret* the conclusions in terms of the original situation.
5. *Validate* the conclusions by comparing them with the situation. Iterate if necessary to improve the model.
6. *Report* the conclusions and the reasoning behind them.

It's important to recognize that in practice, these actions don't often happen in a nice, neat order.

[1] Adapted from IM 9–12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/index.html,

For this first modeling prompt block, there is substantial background teacher guidance on what it means to model, how to set up the classroom for modeling, how to support students around the modeling prompt, and how to interpret the modeling prompt lift analyses.

What are you excited to learn about modeling?

Agenda, Materials, and Preparation

- **Modeling Prompt #1: Evaluating a Sample Response to a Modeling Prompt** (*Recommended*)
  - Modeling Prompt #1 and Sample Response (print 1 copy per student)
- **Modeling Prompt #2: Display Your Data** (*Optional*)
  - Modeling Prompt #2 (print 1 copy per student)
  - Modeling Rubric (print 1 copy per student)

Modeling Prompts Teacher Guidance

Mathematical Modeling Prompt

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

The mathematical modeling prompts and this guidance for how to use them represent an effort to make authentic modeling accessible to all teachers and students.

Organizing Principles about Mathematical Modeling

- The purpose of mathematical modeling in school mathematics courses is for students to understand that they can use math to better understand things in the world that interest them.
- Mathematical modeling is different from solving word problems. It often feels like initially there is not enough information to answer the question. There should be room to interpret the problem. There ought to be a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- It is expected that students have support from their teacher and classmates while modeling with mathematics. It is not a solitary activity. Assessment should focus on feedback that helps students improve their modeling skills.

Preparing for a Modeling Prompt

Ideas for Setting Up an Environment Conducive to Modeling	How to Prepare and Conduct the Modeling Lesson
• Provide plenty of blank whiteboard or chalkboard space for groups to work together comfortably. “Vertical non-permanent surfaces” are most conducive to productive collaborative work. “Vertical” means on a vertical wall, which is better than horizontally on a tabletop, and “non-permanent” means something like a dry erase board, which is better than something like chart paper (Liljedahl 2016). • Ensure that students have easy access to any tools that might be useful for the task. These might include: – supply table containing geometry tools, calculators, scratch paper, graph paper, dry erase markers (ideally a different color for each group member), post-its – electronic devices to access digital tools (like graphing technology, dynamic geometry software, or statistical technology) • Think about how to help students manage the time that is available to work on the task. For example: – Display a countdown timer for intermittent points in the lesson when groups are asked to summarize their progress. – Decide what time to ask groups to transition to writing down their findings in a somewhat organized way (perhaps 15 minutes before the end of the class).	• Decide which version of the prompt students will receive, based on the lift-analysis, timing, and access to data. • Have data ready to share if planning to give it when students ask. • Decide if students will be offered a template for organizing modeling work. • Decide to what extent students are expected to iterate and refine their model. The amount of time available can influence how much time students have to refine their model. If time is short, students may not engage as much in that part of the modeling cycle. WIth more time, it is more reasonable to expect students to iterate and refine their model once or even several times. • Decide how students will report their results. Again, if time is short, this may be a rough visual display on a whiteboard. If more time is available, students might create a more formal report, slideshow, blog post, poster, mockup of an artifact like a letter to a specific audience, smartphone app, menu, or set of policies for a government entity to consider. One way to scaffold this work is to ask students to turn in a certain number of presentation slides: one that states the assumptions made, one that describes the model, and one or more slides with their conclusions or recommendations. • Develop task-specific “look-fors” for each dimension of the provided rubric. What do you anticipate and hope to see in student work?
Organizing Students Into Teams or Groups	**Ways to Support Students While They Work on a Modeling Prompt**
• Mathematical modeling is not a solitary activity. It works best when students have support from each other and their teacher. • Working with a team can make it possible to complete the work in a finite amount of class time. For example, the team may decide it wants to vary one element of the prompt and compute the output for each variation. What would be many tedious calculations for one person could be only a few calculations for each team member. • The members of good modeling groups bring a diverse set of skills and points of view. Create and share a Multiple Abilities List with students. • Scramble the members of modeling teams often, so that students have opportunities to play different roles.	• Coach students on ways to organize their work. • Provide a template to help students organize their thinking. Over time, some groups may transition away from needing to use a template. • Engage students in the *Three Reads* instructional routine to ensure comprehension of the prompt. • Remind students of the variety of tools that are available to them. • If students get stuck or run out of ideas, help move them forward with a question that prompts them to focus on a specific part of the modeling cycle. For example: – “What quantities are important? Which ones change and which ones stay the same?” – “What information do you know? What information would it be nice to know? How could you get that information? What reasonable assumption could you make?” – “What pictures, diagrams, graphs, or equations might help people understand the relationships between the quantities?” – “How are you describing the situation mathematically? Where does your solution come from?” – “Under what conditions does your model work? When might it not work?” – “How could you make your model better? How could you make your model more useful under more conditions?” – “What parts of your solution might be confusing to someone reading it? How could you make it more clear?”

How to Interpret the Provided Lift Analysis of a Modeling Prompt

For most mathematical modeling prompts, different versions are provided. Each version is analyzed along five impactful dimensions that vary the demands on the modeler (OECD 2013). Each of the attributes of a modeling problem is scored on a scale from 0–2. A lower score indicates a prompt with a “lighter lift” for students and teachers: students are engaging in less open, less authentic mathematical modeling. A higher score indicates a prompt with a “heavier lift” for students and teachers: students are engaging in more open, more authentic mathematical modeling. This matrix shows the attributes that are part of our analysis of each mathematical modeling prompt. Though not all the attributes have the same impact on what teachers and students do, for the sake of simplicity, they are all weighted the same when they are averaged.

Attribute		DQ Defining the Question	QI Quantities of Interest	SD Source of Data	AD Amount of Data given	M The Model
Lift	Light lift (0)	Well-posed question	Key variables declared	Data are provided	Modeler is given all the information they need and no more	Model is given in the form of a mathematical representation
	Medium lift (1)	Elements of ambiguity; prompt might suggest ways assumptions could be made	Key variables suggested	Modelers are told what measurements to take or data to look up	Some extra information is given and modeler must decide what is important; or, not enough information is given and modeler must ask for it before teacher provides it	Modeler must sift through lots of given information and decide what is important; or, not enough information is given and modeler must make assumptions, look it up, or take measurements
	Heavy lift (2)	Freedom to specify and simplify the prompt; modeler must state assumptions	Key variables not evident	Modelers must decide what measurements to take or data to look up	Modeler must sift through lots of given information and decide what is important; or, not enough information is given and modeler must make assumptions, look it up, or take measurements	Careful thought about quantities and relationships or additional work (like constructing a scatterplot or drawing geometric examples) is required to identify type of model to use

Each version of a mathematical modeling prompt is accompanied by an analysis chart that looks like this sample:

Attribute	DQ	QI	SD	AD	M	Avg
Lift	0	1	0	0	2	0.6

There are other features of a mathematical modeling prompt that could be varied. In the interest of not making things too complex, there are only five dimensions included in the lift analysis. However, a prompt could be additionally modified on one of these dimensions:

- whether the scenario is posed with words, a highly-structured image or video, or real-world artifacts like articles or authentic diagrams
- presenting example for student to explore before they are expected to engage with the prompt, versus the prompt suggesting that the modeler generate examples or expecting the modeler to generate examples on their own
- whether the prompt makes decisions about units of measure or expects the modeler to reconcile units of measure or employ dimensional thinking
- whether a pre-made digital or analog tool is provided, instructions given for using a particular tool, use of a particular tool is suggested, or modelers simply have access to familiar tools but are not prompted to use them
- whether a mathematical representation is given, suggested, or modelers have the freedom to select and create representations of their own choosing

LESSON

Modeling Prompt #1: Evaluating a Sample Response to a Modeling Prompt

This first modeling prompt of Math 1 is structured differently from others in the course. Instead of responding to a prompt, students evaluate a given sample response to a prompt. The purpose is for students to learn more about what a mathematical modeling prompt is and what a response to a modeling prompt might look like. Modeling with mathematics is a complex set of skills, and evaluating those skills is also complex. Students may not have been asked to create models before. Trying something new when the expectations are unclear can be a stressful experience. By evaluating a sample response to a modeling prompt themselves, students will better understand the expectations for the evaluation of their own responses, and this will help them gain confidence in their ability to succeed at modeling.

Student Task Statement 1 Lift Analysis

Attribute	DQ	QI	SD	AD	M	Avg
Lift	0	0	0	0	0	0

Step 1

- Display Advice on Modeling and encourage students to review this in their Student Workbook.
- Facilitate a discussion around modeling. Share some of the following ideas:
  - Modeling prompts are often expressed in words, but unlike word problems, modeling prompts challenge the modeler to make reasonable assumptions, decide what information is important, ask for or research for more information if needed, think creatively within constraints, and consider the implications of the model.
  - The process of modeling is cyclical, and it does not end by producing a “correct answer.”
  - A mathematical model expresses a simplified relationship in the real world; models can be rough but still useful.
  - Models can often be refined to represent the real-world relationship more accurately.

Advice on Modeling

These are some steps that successful modelers often take and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

Understand the Question
Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.

Refine the Question
If necessary, rewrite the question you are trying to answer so that it is more specific.

Estimate a Reasonable Answer
If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.

Identify Unknowns
- What are the meaningful quantities in this situation? Write them down.
- What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.

Gather Information
Write down any of the unknown information that you find. As you work, organize your information in a way that makes sense to you.

Experiment!
Try different ideas to make progress toward answering your question. If you are stuck, think about:
- Helpful ways to organize the information you have or organize your work
- Questions you *can* answer using the information you have
- Ways to represent mathematical relationships or sets of data (tables, equations, scatter plots, graphs, statistical plots)
- Tools that are available for representing mathematics, both digital and analog

Check Your Reasoning
Do you have a first answer to your question? Great! See if it's reasonable.
- Make sure you can explain what the answer means in terms of the original problem.
- Check your precision: Is your answer overly precise (do you really need all those decimal places)? Not precise enough (were you overly aggressive with your rounding)?

Use and Improve Your Model
- Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used?
- What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account.
- What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them?
- What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

Step 2

- Display the modeling prompt for all to see.

> Two friends, Han and Jada, live 7 miles apart. One Saturday, they decide to meet up somewhere between their houses. They each leave their house at 8 a.m. and travel toward each other. They want to choose a place to meet so that they'll both arrive at the same time. Where could they meet?

- Give students quiet think time to answer these questions:
  - What do I understand about this task?
  - What would I need to know in order to answer this question myself?
- Ask, "What questions do you have about the prompt?" After answering any clarifying questions, ask, "If you were going to respond to this prompt, what would you try?"
- Write down students' ideas for all to see. They don't need to find a complete strategy for answering the prompt. The goals of this discussion are to make sure students understand what the prompt is asking and to put students in the position of the modeler trying to answer the prompt. This will prepare them to understand the sample model when they read it.

Step 3

- Display the rubric that students will use to evaluate the sample model and encourage students to review this in their Student Workbook.
- Provide quiet think time and then invite students to share with a partner something they notice and something they wonder about each of the rubric categories.
- Once students have shared, ask, "What would you like to know that would help you use this rubric?" Students should have a good-enough understanding of the rubric that they can feel comfortable trying to use it, but they may discover the answers to some of their questions themselves as they evaluate the sample model; deep understanding of the rubric is not needed at this time.

Modeling Rubric

Skill	Score			Notes or Comments
	Proficient	Developing	Needs Revisiting	
1. Decide What to Model	• Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate. • Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.	• Assumptions are noted but lacking in justification or difficult to find. • Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.	• No assumptions are stated. • No variables are defined.	
	To improve at this skill, you could: • Ask questions about the situation to understand it better • Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?) • Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)			
2. Formulate a Mathematical Model	• An appropriate model is chosen and represented clearly. • Diagrams, graphs, etc. are clear and appropriately labeled.	Parts of the model are unclear, incomplete, or contain mistakes.	No model is presented, or the presentation contains significant errors.	
	To improve at this skill, you could: • Check your model more carefully to make sure it really fits well • Consider a wider variety of possible models, to find one that fits the situation better • Think about the situation more deeply before trying to find a model • Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.			
3. Use Your Model to Reach a Conclusion	• Solution is relevant to the original problem. • Reader can easily understand the reasoning leading to the solution. • Relevant details are included like units of measure.	Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.	No solution is provided.	
	To improve at this skill, you could: • Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later • Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model • Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions			
4. Refine and Share Your Model	• The model's implications are clearly stated. • The limitations of the model and solution are addressed.	The limitations of the model and solution are addressed but lacking in depth or ignoring key components.	No interpretation of model and solution is provided.	
	To improve at this skill, you could: • Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?" • Be skeptical of your model: What don't you like about it, and what can you do to fix those things? • Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.			

Step 4

- Distribute a copy of the Modeling Prompt and Sample Response and review the Modeling Rubric in the Student Workbook.
- Share with students that responses to modeling prompts can vary widely, but they often contain certain pieces: assumptions, calculations, a mathematical model (stated with an equation or equations, with a graph, with a geometric diagram, or in words), conclusions, and generalizations.
  - Ask students to identify these different sections of the sample model in some way (for example, by outlining them in different colors) before they evaluate it.
- Students should have ample opportunity to discuss the sample response and the degree to which it meets each skill on the rubric, but they do not need to reach a group consensus on how to evaluate the response. Instead, students should be encouraged to justify the way they evaluated the response by writing comments in the "Notes or Comments" boxes in the rubric.

RESPONSIVE STRATEGY

As students work in groups, engage them in the Round Robin routine. In this routine, students will each have a turn sharing their thinking, followed by opportunities for group members to ask each other questions and discuss where there are areas of agreement and disagreement.

Round Robin

Step 5

- After students evaluate the sample response, facilitate a whole-class discussion. The goals of this discussion are to make sure students can interpret the rubric and to prepare them to apply what they've learned from the process of understanding and evaluating the sample response when they respond to a modeling prompt for the first time themselves. Here are some questions for discussion:
  - "What did the modeler do well?"
  - "How could this response be improved?"
  - "What part of the response was easiest to understand? What could have been explained better?"
  - "Which skill should the modeler work on?"
  - "Was there something you disagreed with your group about?"
  - "If you were the modeler's teacher, what advice would you give them?"
  - "When you create your own mathematical models, which of the skills on the rubric do you think will be difficult for you? Which will be easier?"

Step 6

- Introduce students to understanding that modeling is a cycle, and that they should evaluate their own models and then refine them by asking them to work together to improve the sample response based on their evaluation of it.
- After sufficient work time, each group or pair should share their improved version with the class so that everyone can see a variety of ways that the response could be improved. Students could share by presenting to the class, doing a gallery walk, uploading a scan or photo of their work to a shared online space, or by any method that works best for the class.

Step 7

- Provide students time to reflect on their experience with this modeling prompt in their Student Workbook.

What do people mean by “modeling with mathematics”? Here is a prompt that could be approached by modeling, and a sample response to understand and evaluate.

Modeling Prompt #1 and Sample Response

Modeling Prompt

Two friends, Han and Jada, live 7 miles apart. One Saturday, they decide to meet up somewhere between their houses. They each leave their house at 8 a.m. and travel toward each other. They want to choose a place to meet so that they'll both arrive at the same time. Where could they meet?

Sample Student Response

Let's assume we don't have to worry about delays like stop lights or traffic.

Here are some quantities we are working with:

- The speed of each friend is probably miles per hour.
- The distance they each travel (miles)
- The total distance they go (miles) - fixed (7 miles)

To simplify the situation, let's pretend (assume) that they each travel at a constant speed the whole time.

Here are some speeds for different methods of travel (see data sources below):

- Walking 2.5–4 miles per hour
- Driving 25–30 miles per hour
- Biking 10–14 miles per hour

If we assume they each use the same method of transportation and they go about the same speed, they will meet pretty much halfway between their houses. For example, if they both walk at a speed of 3.5 miles per hour, they'd each walk 3.5 miles in an hour and should pick a place to meet up halfway between their houses because 3.5 + 3.5 = 7.

But if they aren't using the same mode of transportation or going the same speed, then they wouldn't meet in the middle. We think the quantities are still the same: total distance covered, distance each friend travels, and speed each friend travels. Also the amount of time it takes them, which is measured in minutes.

Let's say Jada bikes and Han walks. So maybe Jada bikes at 12mph and Han walks 3mph. That means Jada travels 1 mile every 5 minutes, and Han travels .25 miles every 5 minutes. Jada is four times as fast as Han and so they would meet up closer to Han's house because Jada was going faster.

	5	10	15	20	25	30
Han	.25	.50	.75	1	1.25	1.50
Jada	1	2	3	4	5	6

In this scenario, they'll meet in less than 30 minutes because at 30 minutes, they would have traveled 7.5 miles and that is more than the 7 miles between their homes. So, they would meet up sometime between 25 and 30 minutes after they started.

Data Sources

1. https://en.wikipedia.org/wiki/Walking
2. http://infinitemonkeycorps.net/projects/cityspeed/
3. https://www.livestrong.com/article/486666-is-an-average-of-15-miles-per-hour-on-a-bike-good-for-a-beginner/

Modeling Prompt #2: Display Your Data

In this modeling prompt, students will work on displaying a data set that answers a particular question. There are two versions of this prompt: 2A and 2B. In 2A, students determine the question they are interested in learning about, find the necessary data, and display their results. In 2B, students are given data to use, determine a question that can be answered with the data, and display their results. Determine, in advance, which Modeling Prompt (2A or 2B) students will receive, based on the lift-analysis, timing, and access to data.

Student Task Statement 2A Lift Analysis

Attribute	DQ	QI	SD	AD	M	Avg
Lift	2	2	2	2	1	1.8

Student Task Statement 2B Lift Analysis

Attribute	DQ	QI	SD	AD	M	Avg
Lift	2	2	2	1	1	1.6

Step 1 (optional step- review materials as necessary)

- Display and pass out the Advice on Modeling and Modeling Rubric handouts.
- Facilitate a discussion around modeling. Share some of the following ideas:
  - Modeling prompts are often expressed in words, but unlike word problems, modeling prompts challenge the modeler to make reasonable assumptions, decide what information is important, ask or research for more information if needed, think creatively within constraints, and consider the implications of the model.
  - The process of modeling is cyclical, and it does not end by producing a "correct answer."
  - A mathematical model expresses a simplified relationship in the real world; models can be rough but still useful.
  - Models can often be refined to represent the real-world relationship more accurately.
  - Responses to modeling prompts can vary widely, but they often contain certain pieces: assumptions, calculations, a mathematical model (stated with an equation or equations, with a graph, with a geometric diagram, or in words), conclusions, and generalizations.
- Provide time for students to ask clarifying questions. Students should have a good-enough understanding of the rubric, however, deep understanding of the rubric is not needed at this time.

Step 2

- Pass out the pre-determined appropriate blackline master Modeling Prompt #2 (2A or 2B).
- Students can be arranged in groups in advance, they can choose groups based on the questions they are most interested in, or use visibly random grouping.
- **Modeling Prompt 2A**
  - Ask students to think about a question they are interested in for which they would need to gather data in order to answer.
  - Students will choose one of these questions, find data relevant to it, and present their results with a data display or an infographic. If needed, show students examples of infographics or ask them to look up a few examples.
- **Modeling Prompt 2B**
  - Give students a source of data and let them explore it for a few minutes. Have them share observations with their classmates. This task can be further scaffolded by giving students data sources that are easier to work with.
  - Students will brainstorm some questions that could be answered with the given data.
  - They will then choose one of these questions, find data relevant to it, and present their results with a data display or an infographic. If needed, show students examples of infographics or ask them to look up a few examples.

RESPONSIVE STRATEGY

If students are not sure where to begin, it might be helpful for them to think about questions that compare two groups, since they have been doing a lot of work with comparing data sets using measures of center and variability, and this is the kind of data analysis they will be expected to do. For example, in states that have sales tax on food do people have less food security than states with no sales tax on food?

It is possible that students may not be able to find the information they need to answer the question they were originally interested in. Offer them the opportunity to refine and change their question to align with the data available.

Modeling Prompt 2A

Choose a question that you think is interesting and that you do not know the answer to. You will gather data relevant to this question by either doing an experiment or researching available data, so choose a reasonable question.

1. Write your question.
2. What do you predict you will learn from the data?
3. Next, gather your data. Then choose one of these options for displaying your data and answer the questions about it.

   Option 1: Create a display that shows the distribution of the data. Include both measures of center and measures of variability.

   - Why is the data display you selected the best way to summarize your data?
   - What is surprising about the data?
   - Are there any outliers? If so, tell their story. If not, why do you think not?
   - Describe the shape of the distribution.

   Option 2: Use an infographic to summarize what you found.

   - Why is an infographic the best way to summarize your data?
   - What is surprising about the data?
   - What story does your infographic tell?
4. Now that you have seen some data, is there another related question you would like to study or some additional data you would like to collect?

Modeling Prompt 2B

Your teacher will give you some data.

1. What are some questions that you might be able to answer with the given data?
2. Choose one of your questions to try to answer. What do you predict the data will tell you?
3. Look through the data to answer your question. You might have to change your question a little if the given data doesn't answer your question. When you have an answer, choose a way of displaying the data that will help other people understand what you learned.

   Option 1: Create a display that shows the distribution of the data. Include both measures of center and measures of variability.

   - Why is the data display you selected the best way to summarize your data?
   - What is surprising about the data?
   - Are there any outliers? If so, tell their story. If not, why do you think not?
   - Describe the shape of the distribution.

   Option 2: Use an infographic to summarize what you found.

   - Why is an infographic the best way to summarize your data?
   - What is surprising about the data?
   - What story does your infographic tell?
4. Now that you have seen some data, is there another related question you would like to study or some additional data you would like to collect?

Step 3

- Remind students that modeling is a cycle, and that they should evaluate their own models and then refine them, as necessary.
- After sufficient work time, each group or pair should share their displays or infographics with the class. Students could share by presenting to the class, doing a gallery walk, uploading a scan or photo of their work to a shared online space, or by any method that works best for the class.

Step 4

- Provide students time to reflect on their experience with this modeling prompt in their Student Workbook.

RESPONSIVE STRATEGY

As students work in groups, engage them in the Round Robin routine. In this routine, students will each have a turn sharing their thinking, followed by opportunities for group members to ask each other questions and discuss where there are areas of agreement and disagreement.

Round Robin

TEACHER REFLECTION

What were three big wins for this first modeling prompt set of lessons?

What would you do differently next time you facilitate these introductory lessons?

What will you keep in mind for the next set of modeling lessons for this year?

Lesson 15: Representing Situations with Inequalities

PREPARATION

Lesson Goal	Learning Target
• Interpret and write inequalities that represent the constraints in a situation.	• I can write inequalities that represent the constraints in a situation.

Lesson Narrative

Prior to this point, students have worked primarily with equations in one and two variables. In this lesson, they shift their attention to inequalities. Here and in the next two lessons, students interpret, write, and find solutions to inequalities in one variable. The activities here are grounded in contexts, and the ideas build on the work students have done in earlier grades.

The focus of this lesson is on interpreting and writing inequalities that represent the constraints in various situations. As students consider key quantities in a situation, select variables to represent them, and decide whether multiple inequalities (and which ones) are needed, they engage in aspects of mathematical modeling (MP4).

In future lessons, students will use the insights from this lesson to find and interpret the solutions to inequalities that model situations.

In what ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Addressing
NC.6.EE.8: Reason about inequalities by: **a.** Using substitution to determine whether a given number in a specified set makes an inequality true. **b.** Writing an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. **c.** Recognizing that inequalities of the form x > c or x < c have infinitely many solutions. **d.** Representing solutions to inequalities on number line diagrams.	**NC.M1.A-SSE.1:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. **b.** Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*30 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U2.L15 Cool-down (print 1 copy per student)

LESSON

Warm-up: What Do Those Symbols Mean? (*5 minutes*)

Building On: NC.6.EE.8	**Building Towards:** NC.M1.A-CED.1

In this activity, students recall the meaning of inequality symbols ($<$, $>$, $\leq$, and $\geq$) and the meaning of "solutions to an inequality." They are reminded that an inequality in one variable can have a range of values that make the statement true. Students also pay attention to the value that is at the boundary of an inequality and consider whether it is or isn't a solution to an inequality.

Step 1

- Give students 1–2 minutes of quiet work time. Follow with a whole-class discussion.

Student Task Statement

1. Match each inequality to the meaning of the symbol within it.

a. $h > 50$	• less than or equal to
b. $h \leq 20$	• greater than
c. $30 \geq h$	• greater than or equal to

2. Is 25 a solution to any of the inequalities? Which one(s)?
3. Is 40 a solution to any of the inequalities? Which one(s)?
4. Is 30 a solution to any of the inequalities? Which one(s)?

Advancing Student Thinking: On the last inequality ($30 \geq h$), some students may interpret the "greater than or equal to" symbol to mean we are looking for values that are greater than or equal to 30. For these students, clarify that the statement reads "30 is greater than or equal to h," which means h must be less than or equal to 30.

Step 2

- Ask students how they know whether each of the numbers 50, 20, and 30, called the boundary values, is a solution to the inequality. Emphasize that we can test those boundary values the same way we test other values—by checking if they make the statement true.
- Display these equations in one variable for all to see: $h = 50$, $h = 20$, and $30 = h$. Discuss with students how these equations are different from the inequalities in one variable (aside from the fact that the symbols are different). Highlight the idea that there is only one value that could make each equation true, but there is a range of values that can make each inequality true.

DO THE MATH

PLANNING NOTES

Activity 1: Elevator Constraints (*30 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Collect and Display (MLR2)
Addressing: NC.M1.A-SSE.1; NC.M1.A-CED.1

In this activity, students interpret a given equation and inequality and make sense of them in terms of a situation. Additionally, students write equations and inequalities to represent the constraints in a situation. Students identify key quantities and relationships, and think about ways to represent them. In doing so, they engage in an *Aspect of Mathematical Modeling* (MP4).

Step 1

- Have students arrange themselves in pairs or use visibly random grouping.
- Provide students with 2 minutes to independently read the opening scenario and the directions for each row in question 1. After 2 minutes, have students summarize the directions with their partner.
- Answer any lingering questions before prompting them to begin.

Monitoring Tip: Monitor for students who represent the same constraint using different inequalities or equations. For example, to represent the total weight that the elevator could carry, some students may write $w \leq 1,500$, some may write $1,500 \geq w$, and others may write $w < 1,500$ or $w = 1,500$. Ask students with contrasting statements to share their responses later.

***Advancing Student Thinking*:** Students who use a variable for the number of adults and another for the number of children may have trouble accounting for the weight of the gear because it applies to both groups. Calculating the weight of a specific combination of adults and children (with their gear) may help.

Student Task Statement

Scenario: An elevator car in a skyscraper can hold at most 15 people. For safety reasons, each car can carry a maximum of 1,500 kg. On average, an adult weighs 70 kg, and a child weighs 35 kg. Assume that each person carries 4 kg of gear with them.

1. With a partner, follow the steps below to complete this table.
   - Row 1: Use this completed row to guide you in the rows that follow.
   - Row 2: Work together with your partner. Use the equation and variable definitions to explain what constraint the equation represents.

- Rows 3–4: Work independently. Create two equations or inequalities that you can think of that also represent constraints in the scenario. Define the variables you use and explain how your equation or inequality represents the constraint. (Avoid using the variables z, m, or g.)
- Rows 5–6:
  - Take turns sharing your equations or inequalities and variable definitions from rows 3 and 4 with one another. Do not share your explanation. Write down your partner's shared information and make sure your partner has correctly written down what you shared. At this point, the only empty boxes in your table should be explanation boxes.
  - Work independently to provide an explanation for the equation or inequality your partner shared with you, using the variable definitions and referring back to the scenario as needed.
  - After both you and your partner have completed the explanations, share the explanations with one another. If there is disagreement, work together to determine whether adjustments need to be made to the equation or inequality to ensure that it is communicated more clearly. If both you and your partner created the same equation or inequality but had different explanations, determine whether the different explanations communicate the same idea.

Row	Example	Equation/Inequality	Explanation	Variables defined
1	Completed example	$0 \le p \le 15$	The elevator car in a skyscraper can hold between, and including, 0 and 15 people.	p represents the number of people in an elevator car
2	Partially completed example	$w = 70a + 35c + 4a + 4c$		a represents the number of adults, c represents the number of children in an elevator car, and w represents the weight in kilograms that one elevator car can carry.
3	Your example			
4	Your example			
5	Partner's example			
6	Partner's example			

2. Independently, rewrite the six equations and inequalities (from question 1) so that they would work for a different building where:
   - an elevator car can hold at most z people
   - each car can carry a maximum of m kilograms
   - each person carries g kg of gear

1.	4.
2.	5.
3.	6.

Step 2

Invite students to share their equations and inequalities, starting with those that are more concrete (from the first question) and ending with the ones that are more abstract (from the last question).

Using the *Collect and Display* routine, amplify and record any student responses that describe the meaning of each equation or inequality on a display. Call students' attention to the different ways the constraints are represented by language in context and equations or inequalities of different forms.

Emphasize that the same constraints may be accurately represented by statements of different forms. Consider reading aloud the different inequalities that represent the same constraint. For example, if w represents the total weight:

- $w \leq 1,500$ can be read: "The total weight is less than or equal to 1,500 kilograms." or "The total weight is at most 1,500 kilograms."
- $1,500 \geq w$ can be read: "Fifteen hundred kilograms is greater than or equal to the total weight."
- $w < 1,500$ or $w = 1,500$ can be read: "The total weight is less than 1,500 kilograms, or it is equal to 1,500 kilograms."

For constraints that involve multiple quantities, some students may write, for instance, $74a + 39c \leq 1,500$, while others may write $70a + 4a + 35c + 4c \leq 1,500$. Ask students why these expressions are equivalent, encouraging them to use the context in their explanation.

Point out that although a constraint can be written in different ways, writing it using fewer terms may be more convenient and may allow certain insights to be gained about the situation.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The goal of this debrief is to have students reflect on why inequalities may be used to communicate constraints in a given scenario and how precise language can make communication more effective.

Facilitate a discussion using the following questions. Decide whether students should first have an opportunity to reflect on these questions in their workbooks or talk through them with a partner. Choose what questions will be prioritized in the full class discussion.

- "What are some advantages to representing constraints with inequality symbols?" (Compared to written words, inequalities are a simpler and quicker way to describe what is happening in a situation. It is easier to see what values a certain quantity could or could not take when the constraint is written with symbols and numbers.)
- "What are some disadvantages to representing constraints using inequality symbols?" (Unless we know what the variables stand for, we can't be sure about the meaning of an inequality. If we don't recall what the symbols mean or how to read them, we can't access the information.)

PLANNING NOTES

- "Some of your classmates might just be learning to write inequalities to represent constraints. What should they pay attention to?" (specifying the meaning of each variable, not using the same variable to represent different quantities, making sure that the correct symbols are used to represent relationships, using words to read each inequality to make sure that it fully represents a constraint)
- "What are some potential sources of confusion or error they should look out for?" (The words "at most" and "at least" can be confusing, but they imply that the boundary value should be included.)

Student Lesson Summary and Glossary

We have used equations and the equal sign to represent relationships and constraints in various situations. Not all relationships and constraints involve equality, however.

In some situations, one quantity is, or needs to be, greater than or less than another. To describe these situations, we can use inequalities and symbols such as $<, \leq, >$, or $\geq$.

When working with inequalities, it helps to remember what the symbol means, in words. For example:

- $100 < a$ means "100 is less than a."
- $y \leq 55$ means "y is less than or equal to 55," or "y is not more than 55."
- $20 > 18$ means "20 is greater than 18."
- $t \geq 40$ means "t is greater than or equal to 40," or "t is at least 40."
- $25 < z < 36$ means "z is between 25 and 36," or "z is greater than 25, but less than 36."

These inequalities are fairly straightforward. Each inequality states the relationship between two numbers ($20 > 18$), or they describe the limit or boundary of a quantity in terms of a number ($100 < a$).

Inequalities can also express relationships or constraints that are more complex. Here are some examples:

- The area of a rectangle, A, with a length of 4 meters and a width of w meters, is greater than 0 square meters but less than or including 100 square meters..

$$0 < A \leq 100$$
$$0 < 4w \leq 100$$

- To cover all the expenses of a musical production each week, the number of weekday tickets sold, d, and the number of weekend tickets sold, e, must be greater than 4,000.

$$d + e > 4{,}000$$

- Elena would like the number of hours she works in a week, h, to be more than 5 but no more than 20.

$$h > 5$$
$$h \leq 20$$

- The total cost, T, of buying a adult shirts and c child shirts must be less than 150. Adult shirts are \$12 each, and children shirts are \$7 each.

$$T < 150$$
$$12a + 7c < 150$$

In upcoming lessons, we'll use inequalities to help us solve problems.

Cool-down: Grape Constraints (*5 minutes*)

Addressing: NC.M1.A-SSE.1; NC.M1.A-CED.1

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Han has a budget of $25 to buy grapes. Write inequalities to represent the number of pounds of grapes that Han could buy in each situation:

1. Grapes cost $1.99 per pound.
2. Grapes cost $2.49 per pound.
3. Grapes cost \$$c$ per pound.

Student Reflection: Give an example of where you might see the mathematics you did today in the real world.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a recent time from class when your students were confused. What did you do to support them in reasoning about their confusion together as a community of learners?

Practice Problems

1. Tyler goes to the store. His budget is $125. Which inequality represents x, the amount in dollars Tyler can spend at the store?

   a. $x \leq 125$
   b. $x \geq 125$
   c. $x > 125$
   d. $x < 125$

2. Jada is making lemonade for a get-together with her friends. She expects a total of 5 to 8 people to be there (including herself). She plans to prepare 2 cups of lemonade for each person.

   The lemonade recipe calls for 4 scoops of lemonade powder for each quart of water. Each quart is equivalent to 4 cups.

   Let n represent the number of people at the get-together, c the number of cups of water, ℓ the number of scoops of lemonade powder.

   Select **all** the mathematical statements that represent the quantities and constraints in the situation.

   a. $5 < n < 8$
   b. $5 \leq n \leq 8$
   c. $c = 2n$
   d. $\ell = c$
   e. $10 < c < 16$
   f. $10 \leq \ell \leq 16$

3. A doctor sees between 7 and 12 patients each day. On Mondays and Tuesdays, the appointments are 15 minutes. On Wednesdays and Thursdays, they are 30 minutes. On Fridays, they are one hour long. The doctor works for no more than 8 hours a day.

   Here are some inequalities that represent this situation.

   $0.25 \leq y \leq 1$

   $7 \leq x \leq 12$

   $xy \leq 8$

   a. What does each variable represent?

   b. What does the expression xy in the last inequality mean in this situation?

4. Han wants to build a dog house. He makes a list of the materials needed:

   - At least 60 square feet of plywood for the surfaces
   - At least 36 feet of wood planks for the frame of the dog house
   - Between 1 and 2 quarts of paint

   Han's budget is $65. Plywood costs $0.70 per square foot; planks of wood cost $0.10 per foot, and paint costs $8 per quart.

   Write inequalities to represent the material constraints and cost constraints in this situation. Be sure to specify what your variables represent.

5. Solve each problem two ways. First, substitute the given values and solve for the given variable. Then, solve for the given variable and substitute for the given values.[1]

   a. The perimeter formula for a rectangle is $p = 2(l + w)$, where p represents the perimeter, l represents the length, and w represents the width. Calculate l when $p = 70$ and $w = 15$.

   b. The area formula for a triangle is $A = \frac{1}{2}bh$, where A represents the area, b represents the length of the base, and h represents the height. Calculate b when $A = 100$ and $h = 20$.

   (From Unit 2, Lesson 11)

6. Elena has $84, and Priya has $12. How much money must Elena give to Priya so that Priya will have three times as much as Elena? Create an equation to represent the problem, and solve for how much Elena must give Priya.[2]

   (From Unit 2, Lesson 8)

7. The equation $V = \frac{1}{3}\pi r^2 h$ represents the volume of a cone, where r is the radius of the cone and h is the height of the cone. Which equation is solved for the height of the cone?

   a. $h = V - \pi r^2$

   b. $h = \frac{1}{3}\pi r^2 V$

   c. $3V - \pi r^2 = h$

   d. $h = \frac{3V}{\pi r^2}$

   (From Unit 2, Lesson 12)

8. A car is going 65 miles per hour down the highway. Mai wrote the equation $y = 65x$ to match the story, with x representing the time traveled, in hours, and y representing the distance traveled, in miles.

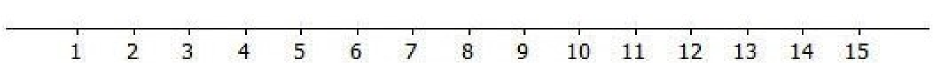
   a. Tyler wrote the equation $x = \frac{y}{65}$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Tyler's equation matches the story.

   b. Lin wrote the equation $y = \frac{x}{65}$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Lin's equation does not match the story.

   (From Unit 2, Lesson 12)

9. A data set consisting of the number of hours each of 40 students watched television over the weekend has a minimum value of 3 hours, a Q1 value of 5 hours, a median value of 6 hours, a Q3 value of 9 hours, and a maximum value of 12 hours.[3]

   a. Draw a box plot representing this data distribution.



   b. What is the interquartile range (IQR) for this distribution? What percent of the students fall within this interval?

   (From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).
[3] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

Lesson 16: Solutions to Inequalities in One Variable

PREPARATION

Lesson Goals	Learning Targets
• Find the solution to a one-variable inequality by reasoning and by solving a related equation and testing values greater than and less than that solution. • Graph the solution to an inequality as a ray on a number line and interpret the solution in context. • Understand that the solution to an inequality is a range of values that make the inequality true.	• I can graph the solution to an inequality in one variable. • I can solve one-variable inequalities and interpret the solutions in terms of the situation. • I understand that the solution to an inequality is a range of values (such as $x > 7$) that makes the inequality true.

Lesson Narrative

In this lesson, students revisit the meaning of the solutions to an inequality in one variable and recall that the **solution set** is a range of values. They also investigate different ways to find the solution set to an inequality—by reasoning about the quantities and relationships in context, by guessing some values, substituting them into the inequality, and checking them to see if they make an inequality true, and by first solving a related equation in one variable. Along the way, students reason abstractly and quantitatively (MP2).

Later, students will use the understanding they build here to solve more sophisticated problems and to find solutions to linear inequalities in two variables.

How is the approach of this lesson similar and different from other ways you have taught these concepts or procedures?

Focus and Coherence

Building On	Addressing
NC.7.EE.4: Use variables to represent quantities to solve real-world or mathematical problems. **b.** Construct inequalities to solve problems by reasoning about the quantities. • Fluently solve multi-step inequalities with the variable on one side, including those generated by word problems. • Compare an algebraic solution process for equations and an algebraic solution process for inequalities. • Graph the solution set of the inequality and interpret in context. **NC.8.EE.7:** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.

Agenda, Materials, and Preparation

- **Bridge** (*5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U2.L16 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Addressing: NC.7.EE.4

The purpose of this bridge is to allow students to practice solving multi-step inequalities. This will help students later in this lesson when they solve more complex multi-step inequalities, including ones with rational numbers.

Student Task Statement

Solve each inequality for x.

1. $3x+3<12.6$
2. $4x-2>22$

DO THE MATH

PLANNING NOTES

Warm-up: Find a Value, Any Value (*5 minutes*)

Addressing: NC.M1.A-REI.3

This warm-up activates what students know about the solutions to an inequality and ways to find the solutions. For the first time, students refer to the solutions as the solution set to the inequality. Throughout their work with one-variable inequalities, students will use the terms "solutions" and "solution set" interchangeably.

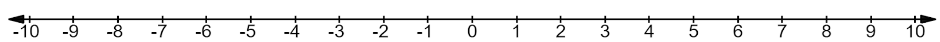
By now, students are likely to have internalized that a solution to an equation in one variable is a value that makes the equation true. The work here makes it explicit that we can extend this understanding of solutions to inequalities in one variable.

Some students may have been taught to solve inequalities the same way we solve equations, with the added rule along the lines of "flip the symbol when dividing or multiplying both sides of an inequality by a negative number." They may or may not have understood why the rule is the way it is. (If a student shares this method, emphasize that they will look at different strategies in the lesson and adopt whichever ways that they can explain or justify.)

If students approach the last question (finding a solution to $7(3-x) > 14$) by performing operations directly on the inequality but neglect to reverse the inequality symbol, they will find solutions that result in false statements. Use this opportunity to point out that we may run into problems with this method. It is not essential to discuss why or to suggest better approaches at this point. There will be other opportunities in this lesson to reason about the solutions and to witness the same issue.

Step 1

Display a number line for all to see that looks like this:



Tell students that, if needed, they could use the number line to help them in reasoning about the inequalities.

Student Task Statement

1. Write one solution to the inequality $y \le 9.2$. Explain what makes a value a solution to this inequality.
2. Write one solution to the inequality $7(3-x) > 14$. Explain your reasoning.

Step 2

- Invite students to share some solutions to the first inequality and explain what it means for a value to be a solution to $y \le 9.2$. Be sure to mention some negative numbers that are solutions.
- Ask each student to quickly mark the one solution they have for $7(3-x) > 14$ on the class number line (from Step 1). (Students could draw a point or put a dot sticker, if available, on the number line.)
- Discuss with students:
  - "How did you know that the value you chose is a solution?" (When substituted for x in the inequality, the value makes a true statement.)
  - "What do you notice about all the points that are on the line?" (They are all to the left side of 1.)
  - "On the number line, we can see that the solutions are values that are less than 1. All these values form the solution set to the inequality. Is there a way to write the solution set concisely, without using the number line and without writing out all the numbers less than 1?" (We can write $x < 1$.)
  - "Does the solution set have anything to do with the solution to the equation $7(3-x) = 14$?" (The solution to the equation is $x = 1$.)

- "Why does the solution set to the inequality $7(3-x) > 14$ involve numbers less than 1?" (The inequality can be taken to say "7 times $3-x$ is greater than 14." For the inequality to be true, $3-x$ must be greater than 2. For $3-x$ to be greater than 2, x must be less than 1. Other partial answers may include: since we are subtracting x, a smaller value for x will make $7(3-x)$ a larger number. If $x=0$, $7(3-x)$ is 21, which is greater than 14.)

- Highlight that we can use a number line to concisely show the solution set to an inequality, but we can also write another inequality that shows the same information.

DO THE MATH	PLANNING NOTES

Activity 1: Off to an Orchard (*15 minutes*)

Instructional Routine: Co-Craft Questions (MLR5)	
Building On: NC.8.EE.7	**Addressing:** NC.M1.A-CED.1; NC.M1.A-REI.3

This activity encourages students to interpret an inequality and its solution set in terms of a situation. A context can help students conceptualize why the solutions to an inequality form a ray on the number line.

The activity also prompts students to think about the solutions to an inequality in terms of a related equation. Here the situation involves choosing between two options with several different valid approaches. The work here encourages students to reason quantitatively and abstractly (MP2), and to make sense of problems and persevere in solving them (MP1).

Step 1

- Facilitate the *Co-Craft Questions* routine. Read the first part of the task statement with the class including the three bullet points, and then give students 1 minute of think time to come up with mathematical questions that could be asked about the situation. Call on just 2–3 students to share a question with the class.

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.

- For the first set of questions, ask one partner to find the cost of going to orchard A and the other partner to find the cost of going to orchard B, and then compare the costs. Before students move on to the second set of questions, pause to hear which option works best for 8, 12, and 30 students.

RESPONSIVE STRATEGIES

Represent the same information through different modalities by using a table. If students are unsure where to begin, suggest that they use a table to help organize the information provided. Guide students in making decisions about what inputs to include in their table. Be sure that they include 17 and some values directly above and below to support their analysis.

Supports accessibility for: Conceptual processing; Visual-spatial processing

M

Monitoring Tip: Monitor the strategies students use to find the solutions to $9(n+3) < 10(n+1)$, and identify students using different approaches.

Students may:

- Try different values of n until the inequality is no longer true.
- Try different numbers higher than 12 (based on their work on the first question) and find that, up to 17 students, the cost to go to orchard B is lower. Beyond 17 students, the cost for orchard A is lower.
- Solve the equation $9(n+3) = 10(n+1)$ to find the number of students at which the costs for both options would be equal. That number is 17. Then, try a higher or lower number to see which side of the equation has a smaller value.
- Reason about the difference in the cost per student and cost for chaperones. The cost per student at orchard A ($9) is $1 lower than at orchard B ($10). But because 3 chaperones are required at orchard A ($27 for 3 chaperones) and only 1 at orchard B ($10 for 1 chaperone), the cost for chaperones is $17 higher at orchard A than at orchard B. So if 17 students go on the trip, the cost would be the same at both places. If more than 17 students go, orchard A would be cheaper.
- Find the solutions to $9(n+3) < 10(n+1)$ by manipulating the inequality to isolate n.

***Advancing Student Thinking*:**

- If students struggle to interpret the meaning of the equation $9(n+3) = 10(n+1)$ and of the inequality $9(n+3) < 10(n+1)$, ask them to think about what each side of the equal sign or the inequality symbol represents.
- Depending on the operations performed, they may end up with an incorrect solution set if they forget to reverse the inequality symbol. (For example, in the final step of solving, they may go from $-1n < -17$ to $n < 17$.) If this happens, bring the issue to students' attention during Step 2.

Student Task Statement

A teacher is choosing between two options for a class field trip to an orchard.

- At orchard A, admission costs $9 per person and 3 chaperones are required.
- At orchard B, the cost is $10 per person, but only 1 chaperone is required.
- At each orchard, the same price applies to both chaperones and students.

1. Determine which orchard would be cheaper to visit for each given number of students:

Number of students	Orchard A	Orchard B
a. 8 students		
b. 12 students		
c. 30 students		

2. To help her compare the cost of her two options, the teacher first writes the equation $9(n+3) = 10(n+1)$, and then she writes the inequality $9(n+3) < 10(n+1)$.

   a. What does n represent in each statement?

   b. In this situation, what does the equation $9(n+3) = 10(n+1)$ mean?

   c. What does the solution to the inequality $9(n+3) < 10(n+1)$ tell us?

   d. Determine the solution set to the inequality $9(n+3) < 10(n+1)$.

Step 3

- Make sure students understand the meaning of the inequality in context and recognize that there are various ways to find the solutions.
- Select previously identified students to share how they found the solution set, in the sequence shown in the Monitor Tip (starting with guessing and checking, and ending with reasoning more structurally). It is not necessary to discuss all the listed strategies, but if the idea of solving a related equation doesn't come up, point it out.
- Explain that one way to think about the solutions to the inequality is by thinking about the solution to a related equation. In this context, the solution to $9(n+3) = 10(n+1)$ gives us the number of students at which it would cost the same to go to either orchard. This is a boundary value for n. On one side of the boundary, the cost of orchard A would be higher. On the other, it would be lower. We can test a value of n that is higher and one that is lower than this boundary value to see which one makes the inequality $9(n+3) < 10(n+1)$ true.
- If a student brings up "flipping the symbol when multiplying or dividing by a negative number" as a strategy, invite them to explain why it works. Emphasize that, in general, it is more helpful and reliable to use reasoning strategies that we understand and can explain. If we use a rule without some idea of how it came about or why it works, we might end up misapplying it (for example, flipping the inequality symbol anytime we see a negative sign, even if we're simply adding or subtracting). If we forget or misremember the rule, we would be stuck or make errors.

DO THE MATH	PLANNING NOTES

Activity 2: Equality and Inequality (*10 minutes*)

Addressing: NC.M1.A-REI.3

The purpose of this activity is to further develop the idea that we can solve an inequality by first solving a related equation. Previously, students reasoned about the solutions to an inequality in context. Here, they transition to solving an inequality without a context.

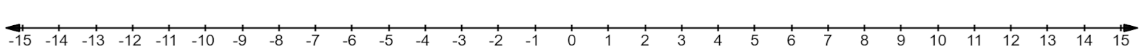
This activity offers another opportunity to point out the trouble with isolating the variable directly from the inequality statement, or with assuming that the solution set can be expressed using the same symbol as in the original inequality. (For example, if in the final step of solving students go from $-24x \leq -48$ to $x \leq 2$, they would end up with the wrong solution set.)

Step 1

- Provide students with quiet time to independently solve the equation in problem 1.

Step 2

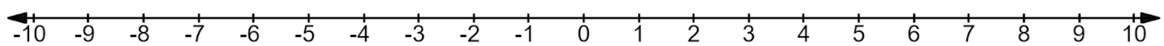
- Assign a number to each student to test if it is a solution to problem 2. Be sure to assign half the class values greater than 2, and the other half values less than 2.
- Display a blank number line such as the following:



- Once students have calculated whether their number is a solution, invite them to record either S (for “Solution”) or N (for “Not a Solution”) under their assigned number.
- Invite students to complete 2a, 2b, and 2c using the display of collected data.

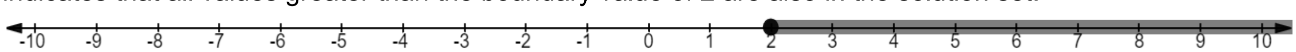
Student Task Statement

1. Solve this equation and check your solution: $-\frac{4(x+3)}{5} = 4x - 12$.
2. Consider the inequality: $-\frac{4(x+3)}{5} \leq 4x - 12$.
   a. Are values less than 2 solutions to the inequality?
   b. Are the values greater than 2 solutions to the inequality?
   c. Choose 2 for x. Is it a solution?
   d. Graph the solution to the inequality on the number line.



Step 3

- On the number line display, represent the solution set, with closed circle and shading, for all to see. Remind students that a closed circle indicates that the boundary value is included in the solution set, and the shading indicates that all values greater than the boundary value of 2 are also in the solution set.



- Emphasize that if an inequality is solved by using a related equation, it is important to make sure that the solution to the equation is correct because that solution gives us a boundary from which the solutions to the inequality can be checked. If the boundary value is off, students may not be able to correctly find the solution set to the inequality.
- Point out that the original inequality involved the “less than or equal to” sign, while the solution set contains numbers that are greater than or equal to 2. Ask students if this is surprising. If needed, remind students that this happened in the warm-up as well.

Are You Ready For More?

Here is a different type of inequality: $x^2 \leq 4$.

1. Is 1 a solution to the inequality? Is 3 a solution? How about -3?
2. Describe all solutions to this inequality. (If you like, you can graph the solutions on a number line.)
3. Describe all solutions to the inequality $x^2 \geq 9$. Test several numbers to make sure your answer is correct.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The goal of this debrief is to have students articulate how solving a related equation can support reasoning about the solution set of a given inequality.

Ask students to respond to the following prompt in their own words and in writing.

- "How does solving the equation $4x-3 = 12(x+3)$ help with solving the inequality $4x-3 \geq 12(x+3)$?" (The solution to the equation $4x-3 = 12(x+3)$ helps me identify the boundary value of the inequality. Once I determine the boundary value, then I can test values less than or greater than the boundary value to identify the solution set to the inequality.)

If time permits, ask students to share their response with a partner, and then invite a student or two to share with the class a particularly clear explanation their partner has written.

PLANNING NOTES

Student Lesson Summary and Glossary

The equation $\frac{1}{2}t = 10$ is an equation in one variable. Its solution is any value of t that makes the equation true. Only $t = 20$ meets that requirement, so 20 is the only solution.

The inequality $\frac{1}{2}t > 10$ is an inequality in one variable. Any value of t that makes the inequality true is a solution. For instance, 30 and 48 are both solutions because substituting these values for t produces true inequalities. $\frac{1}{2}(30) > 10$ is true, as is $\frac{1}{2}(48) > 10$. Because the inequality has a range of values that make it true, we sometimes refer to *all* the solutions as the **solution set**.

Solution set to an inequality: All the values that make the inequality true.

One way to find the solutions to an inequality is by reasoning. For example, to find the solutions to $2p < 8$, we can reason that if 2 times a value is less than 8, then that value must be less than 4. So a solution to $2p < 8$ is any value of p that is less than 4.

Another way to find the solutions to $2p < 8$ is to solve the related equation $2p = 8$. In this case, dividing each side of the equation by 2 gives $p = 4$. This point, where p is 4, is the *boundary* of the solution to the inequality.

To find out the range of values that make the inequality true, we can try values less than and greater than 4 in our inequality and see which ones make a true statement.

Let's try some values less than 4:

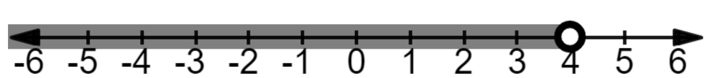
- If $p = 3$, the inequality is $2(3) < 8$ or $6 < 8$, which is true.
- If $p = -1$, the inequality is $2(-1) < 8$ or $-2 < 8$, which is also true.

Let's try values greater than 4:

- If $p = 5$, the inequality is $2(5) < 8$ or $10 < 8$, which is false.
- If $p = 12$, the inequality is $2(12) < 8$ or $24 < 8$, which is also false.

In general, the inequality is false when p is greater than or equal to 4 and true when p is less than 4.

We can represent the solution set to an inequality by writing an inequality, $p < 4$, or by graphing on a number line. The ray pointing to the left represents all values less than 4.



Cool-down: Seeking Solutions (*5 minutes*)

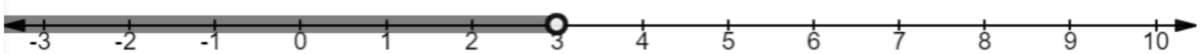
Addressing: NC.M1.A-REI.3

Cool-down Guidance: More Chances
Lesson 17 Activity 1 offers another opportunity to discuss determining if a solution set is greater than or less than a boundary value.

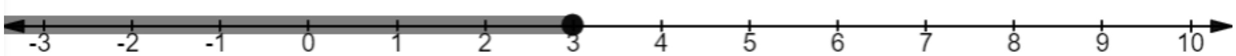
Cool-down

Which graph correctly shows the solution to the inequality $\frac{7x-3}{9} \geq 8 - 2x$? Show or explain your reasoning.

a. 

b. 

c. 

d. 

Student Reflection: Pretend a classmate was absent today. What are three or four key lesson items you would share with them to help them catch up?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What unfinished learning or misunderstandings do your students have about solving inequalities? How did you leverage those misconceptions in a positive way to further the understanding of the class?

Practice Problems

1. Here is an inequality: $\frac{7x+6}{2} \leq 3x + 2$

   Select **all** of the values that are a solution to the inequality.

   a. $x = -3$
   b. $x = -2$
   c. $x = -1$
   d. $x = 0$
   e. $x = 1$
   f. $x = 2$
   g. $x = 3$

2. Find the solution set to this inequality: $2x - 3 > \frac{2x-5}{2}$

   a. $x < \frac{1}{2}$
   b. $x > \frac{1}{2}$
   c. $x \leq \frac{1}{2}$
   d. $x \geq \frac{1}{2}$

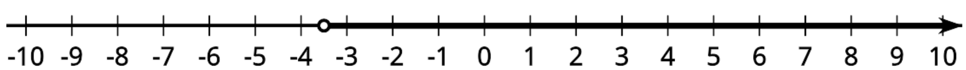
3. Here is an inequality: $\frac{-10+x}{4} + 5 \geq \frac{7x-5}{3}$

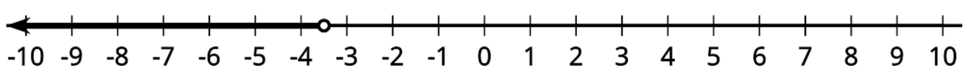
   What value of x will make the two sides equal?

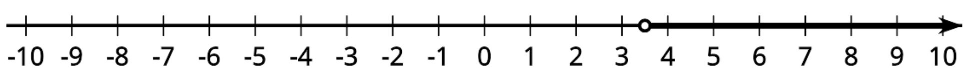
4. Noah is solving the inequality $7x + 5 > 2x + 35$. First, he solves the equation $7x + 5 = 2x + 35$ and gets $x = 6$.

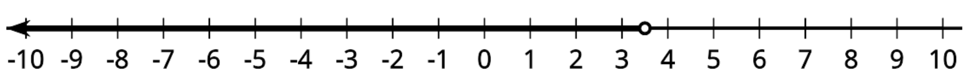
   How does the solution to the equation $7x + 5 = 2x + 35$ help Noah solve the inequality $7x + 5 > 2x + 35$? Explain your reasoning.

5. Which graph represents the solution to $5 + 8x < 3(2x + 4)$?

   a. 

   b. 

   c. 

   d. 

6. Solve the following equations:[1]

   a. $-16 - 6v = -2(8v - 7)$

   b. $2(6b + 8) = 4 + 6b$

   c. $7 - 8x = 7(1 + 7x)$

   d. $39 - 8n = -8(3 + 4n) + 3n$

   (From Unit 2, Lesson 8)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

7. The principal of a school is hosting a small luncheon for her staff. She plans to prepare two sandwiches for each person. Some staff members offer to bring salads and beverages.

   The principal has a budget of $225 and expects at least 16 people to attend. Sandwiches cost $3 each.

   Select **all** of the equations and inequalities that could represent the constraints in the situation, where n is number of people attending and s is the number of sandwiches.

   a. $n \geq 16$
   b. $n \geq 32$
   c. $s < 2n$
   d. $s = 2n$
   e. $3n \leq 225$
   f. $3s \leq 225$

   (From Unit 2, Lesson 15)

8. Students at the college are allowed to work on campus no more than 20 hours per week. The jobs that are available pay different rates, starting from $8.75 an hour. Students can earn a maximum of $320 per week.

   Write at least two inequalities that could represent the constraints in this situation. Be sure to specify what your variables represent.

   (From Unit 2, Lesson 15)

9. Two professional race car drivers have the same average lap times after 50 laps. What does it mean to say that the first driver's lap times have a greater standard deviation than the second driver's lap times?

   (From Unit 1)

10. The solution to $5 - 3x > 35$ is either $x >- 10$ or $-10 > x$. Which solution is correct? Explain how you know.[2]

    (Addressing NC.7.EE.4)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 17: Writing and Solving Inequalities in One Variable

PREPARATION

Lesson Goals	Learning Targets
• Analyze and use the structure in inequalities to determine whether the solution is greater or less than the solution to a related equation. • Write and solve inequalities in one variable to represent the constraints in situations and to solve problems.	• I can analyze the structure of an inequality in one variable to help determine if the solution is greater or less than the solution to the related equation. • I can write and solve inequalities to answer questions about a situation.

Lesson Narrative

This lesson serves two main goals. The first is to prompt students to write and solve inequalities to answer questions about a situation. They consider not only whether the inequalities appropriately model the situations, but also whether there are assumptions that need to be stated and whether the solution sets make sense in context. Along the way, they practice reasoning quantitatively and abstractly (MP2) and engage in aspects of mathematical modeling (MP4).

The second goal is to practice finding the solution set to an inequality by reasoning about its composition and parts. Take $0.5x > 10x$, for example. We can reason that for 0.5 times a number to be greater than 10 times the same number, the number must be negative, so the solution is $x < 0$. Likewise, we can see that all values of x are solutions to $x < x + 5$ because 5 more than a number will always be greater than that number. Students practice looking for and making use of structure (MP7) as they reason about solutions this way.

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.7.EE.4: Use variables to represent quantities to solve real-world or mathematical problems. **b.** Construct inequalities to solve problems by reasoning about the quantities. ● Fluently solve multi-step inequalities with the variable on one side, including those generated by word problems. ● Compare an algebraic solution process for equations and an algebraic solution process for inequalities. ● Graph the solution set of the inequality and interpret in context. **NC.8.EE.7:** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. ● Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. ● Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides. **NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - *Responsive Strategy*: Matching Inequalities and Solutions graphic organizer (print as needed)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U2.L17 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.4; NC.8.EE.7

The purpose of this bridge is for students to practice interpreting the solution of an inequality, especially those involving negative numbers. This connects to the work done in Lesson 16, so students who may need extra practice with this skill can benefit from this bridge task. The work in this task will be useful later in this lesson when students have to make sense of the solutions to inequalities after solving them.

Student Task Statement

Solve each inequality and check your answer using a value that makes your solution true.

1. $-2x < 4$
2. $3x + 5 > 6x - 4$

DO THE MATH	PLANNING NOTES

Warm-up: Dinner for Drama Club (*5 minutes*)

Instructional Routine: Aspects of Mathematical Modeling
Addressing: NC.M1.A-CED.1

In this warm-up, students practice writing an inequality to represent a constraint, reasoning about its solutions, and interpreting the solutions. The work here engages students in *Aspects of Mathematical Modeling*.

To write an inequality, students need to attend carefully to verbal clues so they can appropriately model the situation. The word "budget," for instance, implies that the exact amount given or any amount less than it meets a certain constraint, without explicitly stating this. When thinking about how many pounds of food Kiran can buy, students should also recognize that the answer involves a range, rather than a single value.

Step 1

- Give students quiet work time. Follow with a whole-class discussion.

Student Task Statement

Kiran is getting dinner for his drama club on the evening of their final rehearsal. The budget for dinner is $60.

Kiran plans to buy some prepared food from a supermarket. The prepared food is sold by the pound, at $5.29 per pound. He also plans to buy two large bottles of sparkling water at $2.49 each.

1. Represent the constraints in the situation mathematically. If you use variables, specify what each one means.
2. How many pounds of prepared food can Kiran buy? Explain or show your reasoning.

Step 2

Ask students to share out one or more inequalities that appropriately model the situation. Then, focus the discussion on the solution set. Possible questions for discussion:

- "What strategy did you use to find the number of pounds of prepared food Kiran could buy?"
- "Does Kiran have to buy exactly 10.4 pounds of prepared food?" (No.) "Can he buy less? Why or why not?" (Yes. He can buy any amount as long as the cost of the food doesn't exceed $55.02. This means he can buy up to 10.4 pounds.)
- "What is the minimum amount he could buy?" (He could buy 0 pounds of food, but this wouldn't make sense if his goal is to feed the club members.)

DO THE MATH	PLANNING NOTES

Activity 1: Different Ways of Solving (*15 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Building On: NC.M1.A-REI.1	**Addressing:** NC.M1.A-REI.3

This activity highlights some ways to decide whether the solution set to an inequality is greater than or less than a particular boundary value (identified by solving a related equation).

Students engage in the *Compare and Connect* routine as they are presented with two approaches to solving an inequality. Both characters (Priya and Andre) took similar steps to solve the related equation, which gave them the same boundary value. But they took different paths to decide the direction of the inequality symbol. Below are the two different paths:

1. Testing a value on both sides of the boundary value. This is a familiar strategy given previous work.
2. Analyzing the structure of the inequality. This strategy is less familiar and thus encourages students to make sense of problems and persevere in solving them (MP1). It is also a great opportunity for students to practice looking for and making use of structure (MP7).

RESPONSIVE STRATEGIES

Display or provide copies of Priya's work and Andre's work on separate displays. Write Priya and Andre's descriptive phrases (as given on the student-facing materials) in contrasting colors/texts, so students can more easily sort and process information. Some students may benefit from time to read and interpret each strategy one at a time.

Supports accessibility for: Conceptual processing; Memory

Expect students to need some support in reasoning structurally, as it is something that takes time and practice to develop.

Step 1

- Have students arrange themselves in pairs or use visibly random grouping. They will need a few minutes of quiet time to make sense of what Andre and Priya have done, and then time to discuss their thinking with their partner.

Step 2

- Pause for a class discussion before students move to the second set of questions and try to solve inequalities using Andre and Priya's methods.
- Select students to share their analyses of Andre and Priya's work. Make sure students can follow Priya's reasoning and understand how Priya decided on $x < 3$ by focusing her comparison on $4x + 3$ and $18 – x$.
- As students work, make note of the different ways students approach the task.

***Advancing Student Thinking*:** Students who perform procedural steps on the inequality may find incorrect answers. For instance, in the third inequality, they may divide each side by -9 and arrive at the incorrect solution $x < -4$. Encourage these students to check their work by substituting numbers into the original inequality.

Student Task Statement

Andre and Priya used different strategies to solve the following inequality but reached the same solution.

$$2(2x+1.5) < 18 - x$$

1. Make sense of each strategy until you can explain what each student has done.

Andre

$$\begin{aligned} 2(2x+1.5) &= 18-x \\ 4x+3 &= 18-x \\ 4x-15 &= -x \\ -15 &= -5x \\ 3 &= x \end{aligned}$$

Testing to see if $x = 4$ is a solution:

$$\begin{aligned} 2(2\cdot 4+1.5) &< 18-4 \\ 2(9.5) &< 14 \\ 19 &< 14 \end{aligned}$$

The inequality is false, so 4 is not a solution. If a number greater than 3 is not a solution, the solution must be less than 3, or $3 > x$.

Priya

$$\begin{aligned} 2(2x+1.5) &= 18-x \\ 4x+3 &= 18-x \\ 5x+3 &= 18 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

In $4x+3=18-x$, there is $4x$ on the left and $-x$ on the right.

If x is a negative number, $4x+3$ could be positive or negative, but $18-x$ will always be positive.

For $4x+3<18-x$ to be true, x must include negative numbers or x must be less than 3.

2. Here are four inequalities.

a. $\frac{1}{5}p > -10$

b. $4(x+7) \leq 4(2x+8)$

c. $-9n < 36$

d. $\frac{c}{3} < -2(c-7)$

Work with a partner to decide on at least two inequalities to solve. Solve one inequality using Andre's strategy (by testing values on either side of the given solution), while your partner uses Priya's strategy (by reasoning about the parts of the inequality). Switch strategies for the other inequality.

Are You Ready For More?

Using positive integers between 1 and 9, and each positive integer at most once, fill in values to get two constraints so that $x = 7$ is the only integer that will satisfy both constraints at the same time.

$$\square x + \square < \square x + \square$$

$$\square x + \square > \square x + \square$$

Step 3

- Select as many students as time permits to share how they used a strategy similar to Priya's to determine the solution set to each inequality. There may be more than one way to reason structurally about a solution set. Invite students who reason in different ways to share their thinking. Record and display their thinking for all to see.
- Students should recognize that the solution set to each inequality should be the same regardless of the reasoning method used.

DO THE MATH	PLANNING NOTES

Activity 2: Matching Inequalities and Solutions (*10 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3) - Responsive Strategy
Addressing: NC.M1.A-REI.3

This activity gives students additional practice in reasoning about the solutions to inequalities without a context. Students can match the inequalities and solutions in a variety of ways—by testing different values, by solving a related equation and then testing values on either side of that solution, or by reasoning about the parts of an inequality or its structure.

This is the first time in the course that students might participate in a *Critique, Correct, Clarify* routine.

CRITIQUE, CORRECT, CLARIFY	***What Is This Routine?*** Students are given a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The writing is an incorrect, incomplete, or unclear “first draft” written argument or explanation, and students’ job is to improve the writing by correcting any errors, clarifying meaning, and adding explanation, justification, or details. The routine begins with a brief critique of the first draft in which the teacher elicits two or three ideas from students to identify what could use improvement; students then individually write second drafts, and finally, the teacher scribes as two or three students read their second drafts aloud. ***Why This Routine?*** *Critique, Correct, Clarify* (MLR3) prompts student reflection, fortifies output, and builds students’ meta-linguistic awareness. The final step of public scribing creates an opportunity to invite all students to help edit a final draft together, so that it makes sense to more people. Teachers can demonstrate with meta-think-alouds and press for details when necessary.

Step 1

- Ask students to match the inequality to the graph that represents its solution.
- Monitor for students who determine the range by analyzing the structure of the inequality. Invite these students to share their reasoning during class discussion later.
- As students work, make note of any common challenges or errors so they can be addressed.

RESPONSIVE STRATEGY

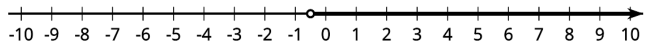
Give students the graphic organizer with a fictitious student's "first draft" work to analyze, reflect on, and improve. Ask students to first identify any parts of the response that are unclear, incomplete, or incorrect by annotating the work. Invite students to share their ideas about what needs improvement with a partner. Then, ask students to create a "second draft" that is more clear, more complete, and more correct by correcting any errors, clarifying meaning, and adding explanation, justification, or details. Finally, ask students to share their second drafts with each other by reading them aloud, and reflecting on their improvements to the "first draft."

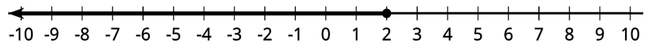
Critique, Correct, Clarify (MLR3)

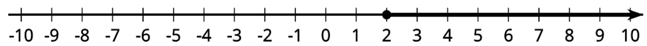
Student Task Statement

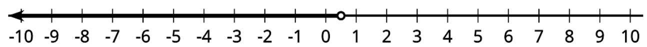
Match each inequality to a graph that represents its solutions. Be prepared to explain or show your reasoning.

1. $5x + 4 \geq 7x$
2. $8x - 2 < -4(x - 1)$
3. $\frac{4x-1}{3} > -1$
4. $\frac{12}{5} - \frac{x}{5} \leq x$

a. 

b. 

c. 

d. 

Are You Ready For More?

Diego's goal is to walk more than 70,000 steps this week. The mean number of steps that Diego walked during the first 4 days of this week is 8,019.

a. Write an inequality that expresses the mean number of steps that Diego needs to walk during the last 3 days of this week to walk more than 70,000 steps. Remember to define any variables that you use.

b. If the mean number of steps Diego walks during the last 3 days of the week is 12,642, will Diego reach his goal of walking more that 70,000 steps this week?

Step 2

- Ask the students who used different strategies—especially those who made use of the structure of the inequalities—to share their thinking. If no students found solution sets by thinking about the features of the inequalities, demonstrate the reasoning process with one or two examples. For instance, to solve the inequality $\frac{4x-1}{3} > -1$
  - After finding $x = -\frac{1}{2}$ as the solution to the related equation $\frac{4x-1}{3} = -1$, we can reason that as x gets smaller, $\frac{4x-1}{3}$ is also going to get smaller.
  - For that expression to be greater than -1, x will have be to greater than $-\frac{1}{2}$.
- If time permits, ask students to choose a different inequality and try reasoning this way about its solution.

DO THE MATH	PLANNING NOTES

Lesson Debrief *(5 minutes)*

In this lesson, students analyze the structure of inequalities and use that structure to determine solutions. They also write and solve inequalities in one variable to represent constraints of a situation.

Display the following inequalities. Remind students that they are examples of inequalities written and solved in the lesson.

$5.29p + 4.98 \leq 60$ and $5.29p + 4.98 \geq 0$

$2(2x + 1.5) < 18 - x$

$\frac{1}{5}x > -10$

Next, tell students that they will see some statements about writing and solving inequalities. Their job is to decide whether they agree or disagree with each statement and be prepared to defend their response. (One way to collect their responses is by asking them to give a discreet hand signal.)

Display the following statements—one at a time—for all to see. After students indicate their agreement or disagreement, select a student from each camp to explain their reasoning. Then invite others who are not convinced by the reasoning to offer a counterexample or an alternative view.

- We can usually represent the constraints in a situation with a single inequality. (Disagree. Sometimes multiple inequalities are needed, depending on what's happening in the situation.)
- The only way to check the solutions to an inequality is to see if they make sense in a situation. (Disagree. We can also check by substituting some values in the solution set back into the inequality to see if they make the inequality a true statement.)
- The only way to find the solutions to an inequality in one variable is by testing different values for the variable and seeing which ones work. (Disagree. We can also reason about the solutions in context, solve a related equation and test a higher and lower value, or use the structure of the inequality.)
- To express the solutions to an inequality in one variable, we always use the same inequality symbol as in the original inequality. (Disagree. For example, the solution set to the inequality $-2x < 8$ is $x > -4$. Additionally, there are different ways to express a solution set. For example, $x < 3$ or $3 > x$ both represent the solution set to $2(2x + 1.5) < 18 - x$.)

Make sure students see that there are reasons for disagreeing with each of these statements and can articulate some of the reasons.

PLANNING NOTES

Student Lesson Summary and Glossary

Writing and solving inequalities can help us make sense of the constraints in a situation and solve problems. Let's look at an example.

Clare would like to buy a video game that costs $\$130$. She has saved $\$48$ so far and plans on saving $\$5$ of her allowance each week. How many weeks, w, will it be until she has enough money to buy the game? To represent the constraints, we can write $48 + 5w \geq 130$. Let's reason about the solutions:

- Because Clare has $\$48$ already and needs to have at least $\$130$ to afford the game, she needs to save at least $\$82$ more.
- If she saves $\$5$ each week, it will take at least $\frac{82}{5}$ weeks to reach $\$82$.

- $\frac{82}{5}$ is 16.4. Any time shorter than 16.4 weeks won't allow her to save enough.
- Assuming she saves $\$5$ at the end of each week (instead of saving smaller amounts throughout a week), it will be at least 17 weeks before she can afford the game.

We can also reason by writing and solving a related equation to find the boundary value for w, and then determine whether the solutions are less than or greater than that value.

$$\begin{aligned} 48+5w &= 130 \\ 5w &= 82 \\ w &= \frac{82}{5} \\ w &= 16.4 \end{aligned}$$

- Substituting 16.4 for w in the original inequality gives a true statement. (When $w=16.4$, we get $130 \geq 130$.)
- Substituting a value greater than 16.4 for w also gives a true statement. (When $w=17$, we get $133 \geq 130$.)
- Substituting a value less than 16.4 for w gives a false statement. (When $w=16$, we get $128 \geq 130$.)
- The solution set is therefore $w \geq 16.4$.

Sometimes the structure of an inequality can help us see whether the solutions are less than or greater than a boundary value. For example, to find the solutions to $3x > 8x$, we can solve the equation $3x = 8x$, which gives us $x = 0$. Then, instead of testing values on either side of 0, we could reason as follows about the inequality:

- If x is a positive value, then $3x$ would be less than $8x$.
- For $3x$ to be greater than $8x$, x must include negative values.
- For the solutions to include negative values, they must be less than 0, so the solution set would be $x < 0$.

Cool-down: How Many Hours of Work? (*5 minutes*)

Addressing: NC.M1.A-CED.1; NC.M1.A-REI.3

Cool-down Guidance: More Chances
Students will have more opportunities to write and solve inequalities in Unit 3.

Cool-down

Lin's job pays $8.25 an hour. She also gets paid $10 per week to cover uniform cleaning. To meet her budget, Lin needs to be paid at least $175 per week.

1. Represent this situation mathematically. If you use variables, specify what each one means.
2. How many hours per week does Lin need to work to meet her budget? Explain or show your reasoning.

Student Reflection:

How do you feel about your level of input during class today (verbal or non-verbal)? What is something you'd like to do differently in the future?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on whose thinking was heard today. Reflect on whose thinking was not heard but could have enriched the conversations. What prompts or structures might better enable the latter to share their voices and reasoning?

Practice Problems

1. Solve $2x < 10$. Explain how to find the solution set.

2. Lin is solving the inequality $15 - x < 14$. She knows the solution to the equation $15 - x = 14$ is $x = 1$.

   How can Lin determine whether $x > 1$ or $x < 1$ is the solution set to the inequality?

3. A cell phone company offers two texting plans. People who use plan A pay 10 cents for each text sent or received. People who use plan B pay 12 dollars per month, and then pay an additional 2 cents for each text sent or received.

   a. Write an inequality to represent the fact that it is cheaper for someone to use plan A than plan B. Use x to represent the number of texts they send.

   b. Solve the inequality.

4. Clare made an error when solving $-4x+3<23$.

   Describe the error that she made.

$$\begin{aligned} -4x+3 &< 23 \\ -4x &< 20 \\ x &< -5 \end{aligned}$$

5. Solve $-4+2t-14-18t>-6-100t$, for t in two different ways.[1]

6. Given this kinetic energy formula $K=\frac{p^2}{2m}$,

   a. Solve for m when $K=75$ and $p=10$.

   b. Rearrange the formula to solve for m.

   (From Unit 2, Lesson 11)

7. Solve for x:

   a. $ax+3b=2f$

   b. $rx+h=sx-k$

   c. $3px=2q(r-5x)$

   (From Unit 2, Lesson 11)[2]

8. What is the solution set of the inequality $\frac{x+2}{2}\geq -7-\frac{x}{2}$?

   a. $x\leq -8$

   b. $x\geq -8$

   c. $x\geq -\frac{9}{2}$

   d. $x\geq 8$

   (From Unit 2, Lesson 16)

9. Here are statistics for the length of some frog jumps in inches:

   - The mean is 41 inches.
   - The median is 39 inches.
   - The standard deviation is about 9.6 inches.
   - The IQR is 5.5 inches.

   How does each statistic change if the length of the jumps are measured in feet instead of inches?

   (From Unit 1)

10. Solve the inequality and check your answer using a value that makes your solution true.

    $-3(x+1)\geq 13$

    (Addressing NC.7.EE.4)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

Lesson 18: Post-Test Activities

PREPARATION

Lesson Goals	Learning Targets
• Communicate high expectations for all students. • Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings.	• I understand the reasoning for and will strive to meet the expectations communicated by my teacher. • I know my classmates and can recognize the value I will add to this classroom community.

Lesson Narrative

This lesson, which should occur after the Unit 2 End-of-Unit Assessment, allows for students to reflect on the unit, share feedback, conference with the teacher, and engage in activities that support the work of the upcoming unit. Note: Activity 3 is required prior to students engaging in Unit 3, Lesson 1, Activity 2.

Gathering student feedback is a powerful and strategic way to learn about students and improve instructional practices. It also creates student and family buy-in and centers students as decision makers and problem solvers in their own learning.

What do you hope to learn about your students during this lesson?

Agenda, Materials, and Preparation

- **Activity 1** (*20 minutes*)
  - End-of-Unit 2 Student Survey (print 1 copy per student)
- **Activity 2** (*15 minutes*)
  - Take Turns card sort (print 1 copy per every 2 students and cut up in advance)
- **Activity 3** (*10 minutes*)
  - Access to technology that can access www.desmos.com/calculator

LESSON

Activity 1: End-of-Unit 2 Student Survey (*20 minutes*)

The End-of-Unit 2 Student Survey is a critical opportunity for teachers to gather low-stakes, non-evaluative feedback to support equity and instructional pedagogy. The survey is also highly beneficial for students as it is designed to encourage self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Provide students a chance to quietly and independently complete this survey after they complete their testing.

One-on-One Conferences

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students complete their surveys and are engaging in Activity 2 and 3. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end-of-unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

Activity 2: Graphs, Tables, Equations, and Situations[1] (*15 minutes*)

Instructional Routines: Card Sort; Take Turns	
Building On: NC.8.F.1; NC.8.F.2	**Building Towards:** NC.M1.A-CED.2; NC.M1.A-CED.3; NC.M1.A-REI.10

In this activity, students get a chance to practice applying their skills at connecting multiple representations of the same linear relationship.

They can use the structure of the *Card Sort* to check their thinking and make sense of the concepts they are practicing since wrong matches will make some piles uneven, or a representation that seems to match one representation in a pile may not match another.

The practice will pay off for the upcoming work of graphing in Unit 3.

Step 1

- Demonstrate how to set up and find matches. Choose a student to be your partner. Mix up the cards and place them face up. Point out that the cards contain either a table, an equation, a situation, or a graph. Select two styles of cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Step 2

- Have students arrange themselves in pairs, or use visibly random grouping, and distribute a set of cut-up slips to each group.
  - Ask students to *Take Turns*: the first partner identifies two cards that match and explains why they think they are a match, while the other listens and works to understand. When both partners agree on the match, they switch roles.

[1] Adapted from IM 9–12 Math Algebra 1 Support Course, Unit 2, Lesson 5 https://curriculum.illustrativemathematics.org/HS/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Student Task Statement

1. Take turns with your partner to match a table, a situation, an equation, and a graph. On your turn, you only need to talk about two cards. Eventually all the cards will be sorted into groups of four cards (an equation, situation, table, and graph).
2. For each match that you find, explain to your partner how you know it's a match. Ask your partner if they agree with your thinking.
3. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Activity 3: Orientation to Graphing Equations with Desmos (*10 minutes*)

Demonstrate how to use Desmos to create and view graphs of equations. Ask students to explore how to enter equations, adjust the graphing window, and plot a point.

- To orient students to graphing on Desmos, provide students with the following instructions go to desmos.com, click "Start Graphing"

Student Task Statement

1. Look for these features:
   a. On the left: blank rows for listing expressions or equations
   b. On the right: a blank coordinate plane
   c. At the bottom: a keyboard
2. Experiment with these actions:
   a. Figure out how to hide the keyboard and the list of expressions. Then, show both features again.
   b. In a blank row on the left, type $y = 2x + 3$. Notice that a graph appears.
   c. Click the y-intercept of the graph to reveal its coordinates.
   d. Click elsewhere on the graph to see the coordinates of the clicked point.
   e. Drag the point along the graph to see how the coordinates change.
3. Follow these instructions:
   a. Delete the first equation and type $y = 100x + 200$.
   b. Can you see the y-intercept? If not, click the button with a "-" sign (on the right side of the graphing window) to zoom out. Repeat until the y-intercept is visible.
   c. Does it look like the graph overlaps with the vertical axis? If so, click the wrench button in the upper right corner.
   d. Experiment with the scales for the x- and y-axes until the graph seems more useful and the intercepts can be seen more clearly. (For example, these boundaries produce a helpful graphing window: $-10 < x < 10$ and $-50 < y < 250$.)
   e. In a blank row on the left, type $(-1, 100)$. Do you see a point plotted? Click this point to reveal its coordinates, or check the "Label" box that appears in the expression list.

TEACHER REFLECTION

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician throughout this work.

List ways you have seen yourself grow as a teacher.

What will you continue to do, and what will you improve upon, in Unit 3?